CSE 632: Analysis of Algorithms II **Fall 2017**

Lecture 8 (09/22/2017): Linear Programming

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8.1 Set Coverage Problem

Problem :

$$\min \sum_{i=1}^{m} y_i \quad \text{s.t.}$$

$$\sum_{i:j\in S_i} y_i \geq 1, \forall j \in [n]$$

$$y_i \geq 0, \forall i \in [m]$$

$\widetilde{y_i} = 0$ for every $i \in [m]$

Algorithm 1 Algorithm for Set Cover LP

do
Choose i^* according to the following distribution:
$Pr[i^* = i] = \frac{y_i}{\sum_{i'=1}^{m} y_{i'}}$
let $\widetilde{y_{i^*}} \leftarrow \widetilde{y_{i^*}} + 1$
until all elements are covered by $\{S_i : \widetilde{y_i} \geq 1\}$

Lemma 8.1

$$\mathbb{E}[\widetilde{x_j}] \geq (1 - \frac{1}{e})x_j$$

$$\Rightarrow \mathbb{E}[\sum_{j=1}^{n} \widetilde{x_j}] \geq (1 - \frac{1}{e}\sum_{j=1}^{n} x_j)$$

Proof:

$$\mathbb{E}[\widetilde{x_j}] = 1 - (1 - \frac{\sum_{i:j\in S_i} y_i}{\sum_{i=1}^{m} y_i})^k \geq 1 - (1 - \frac{\sum_{i:j\in S_i} y_i}{k})^k$$
$$\geq 1 - (1 - \frac{x_j}{k})^{kln(n^2)}$$

$$\geq 1 - e^{x_j ln(n^2)}$$

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Need :

$$\frac{1 - e^{x_j ln(n^2)}}{x_j} \geq 1 - \frac{1}{e^{ln(n^2)}} = 1 - \frac{1}{n^2}$$

8.2 $f - approxiamtion$ for Set Coverage Problem

Define : $f = \max_j |\{i : j \in S_i\}|$

Goal : obtain an f-approximation for set cover

This problem is similar to vertex cover problem :

Given $G = (V, E)$, V : sets, E : elements
Choose a minimum - size set $U \subseteq V$ s.t. $\forall (u, v) \in E, \{u, v\} \cap U \neq \emptyset$

LP rounding algorithm that gives $f - approxiamtion$ for set-cover problem.

$$\widetilde{y_i} = \begin{cases} 1 & \text{if } y_i \geq \frac{1}{f} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{return}\{S_i : \widetilde{y_i} = 1\}$$

$$\widetilde{y_i} \leq f y_i$$

$$\sum_{i=1}^{m} \widetilde{y_i} \leq f \sum_{i=1}^{m} y_i$$

This is also works for weighted problem:

$$\sum_{i=1}^{m} w_i \widetilde{y_i} \leq f \sum_{i=1}^{m} w_i y_i$$