Computer Communications 35 (2012) 207–220

Contents lists available at SciVerse ScienceDirect

Computer Communications

journal homepage: www.elsevier.com/locate/comcom

Cluster size optimization in sensor networks with decentralized cluster-based protocols

Navid Amini *, Alireza Vahdatpour, Wenyao Xu, Mario Gerla, Majid Sarrafzadeh

Computer Science Department, University of California, Los Angeles, United States

ARTICLE INFO

Article history:
Received 16 November 2010
Received in revised form 4 June 2011
Accepted 12 September 2011
Available online 17 September 2011

Keywords:
Sensor networks
Energy-efficiency
Communication protocols
Power control
Network lifetime

ABSTRACT

Network lifetime and energy-efficiency are viewed as the dominating considerations in designing cluster-based communication protocols for wireless sensor networks. This paper analytically provides the optimal cluster size that minimizes the total energy expenditure in such networks, where all sensors communicate data through their elected cluster heads to the base station in a decentralized fashion. LEACH, LEACH-Coverage, and DBS comprise three cluster-based protocols investigated in this paper that do not require any centralized support from a certain node. The analytical outcomes are given in the form of closed-form expressions for various widely-used network configurations. Extensive simulations on different networks are used to confirm the expectations based on the analytical results. To obtain a thorough understanding of the results, cluster number variability problem is identified and inspected from the energy consumption point of view.

1. Introduction

Clustered wireless sensor networks have been envisioned to enable numerous environmental monitoring applications. These networks, besides their energy savings, possess other desirable objectives such as load balancing, fault-tolerance, and increased connectivity. In such networks, nodes organize themselves into multiple clusters either centrally or in a distributed manner. The data, which is the observation of the environment by the sensor, is transmitted to the base station (BS) through the cluster head nodes.

As the ratio of energy consumption for communicating one bit over processing it is in the range of 1000–10000 [1], the dominant source of energy loss in these battery-powered wireless sensor networks is the radio subsystem [2]. As a result, designing an energy-efficient communication protocol is a must. Cluster-based communication protocols have significant savings in total energy consumption of a sensor network. In these protocols, creation of clusters and assigning special tasks to cluster heads can greatly contribute to overall system scalability, lifetime, and bandwidth efficiency [3]. Further, cluster-based routing is an efficient way to lower energy consumption within a cluster by performing data aggregation and therefore, decreasing the number of transmitted messages to the BS together with eliminating data redundancy. Even in the case of networks with densely deployed nodes, communication protocols must provide long-term coverage for the network as well as low and balanced energy consumption of each node. These requirements are closely related to the lifetime and dependability of wireless sensor networks; low energy levels not only are an indication of performance and longevity of sensor networks, but they can influence sensor readings in various ways and result in less reliable or even faulty data as well [4].

A great number of cluster-based communication protocols have been proposed in the literature (see, e.g., LEACH [5,6], LEACH-C [6], LEACH-Coverage [7], DBS [8], X-LLC [9], FTPASC [10]), among which LEACH, LEACH-Coverage and DBS are categorized as decentralized schemes. In these protocols, sensor nodes self-organize themselves into a number of clusters in an autonomous manner and without any centralized support from a certain node which is at odds with aiming to establish a scalable communication protocol.

Exploiting both small and large clusters, the sensor network will end up wasting a large amount of energy. In extreme cases, when the cluster size is very large (e.g., one cluster in the whole network), most of sensor nodes have to transmit their data very far to the reach the cluster head, draining their energy. Similarly, when the cluster size is very small (e.g., clusters of size one) the energy savings owing to the data aggregation will be reduced to a great extent.

In designing centralized cluster-based protocols (e.g., LEACH-C), the problem of finding the optimal cluster size is not a significant matter of concern. This can be attributed to the presence of a powerful controller node which is responsible for periodically

* Corresponding author. Tel.: +1 310 696 9101.
E-mail addresses: amini@cs.ucla.edu (N. Amini), alireza@cs.ucla.edu (A. Vahdatpour), wxu@ee.ucla.edu (W. Xu), gerla@cs.ucla.edu (M. Gerla), majid@cs.ucla.edu (M. Sarrafzadeh).

0140-3664/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.comcom.2011.09.009

determining the optimal cluster size based on the current state of the network and adapting the network accordingly. Whereas, in decentralized cluster-based protocols, there is no global updates in terms of network parameters (e.g., cluster size) and one has to derive such parameters prior to network operation by making reasonable assumptions regarding the distribution of the nodes across the network.

Despite the fact that many researchers require a mathematical framework to evaluate the energy-efficient cluster size (either to improve the lifetime or to fairly compare against other counterpart cluster-based schemes), none of the existing literatures provides such a framework for different network configurations. Furthermore, current decentralized cluster-based protocols (LEACH, LEACH-Coverage, DBS) do not thoroughly consider the issue of optimal cluster size that brings about the maximum lifetime of the network. There are a few articles that either customize their solution for few network configurations [11–13], or they present non-closed-form solutions to this problem [6,14,15]. Further, most of the existing literatures excessively simplify the energy consumption model of the radio subsystem by neglecting the energy dissipated at the receiver side within each wireless communication.

Considering various network configurations, in this work, we explore the effect of cluster size (or equivalently, the number of clusters) on the total energy consumption. Correspondingly, the optimal cluster size that yields the minimum total energy consumption is analytically obtained and presented in the form of closed-form expressions. Extensive simulations have been carried out in our custom simulator in MATLAB to monitor the energy consumption of a network that employs LEACH, LEACH-Coverage, or DBS as its communication protocol. The simulations results are favorably in accordance with the analytical closed-form results.

Generally, the contribution of this paper includes analyzing the optimal cluster size for decentralized sensor networks which leads to minimum total energy expenditure of the network, verification of the analytical results through comprehensive simulations, and finally identification of the cluster number variability as one of the issues that leads to energy loss. More specifically, it is shown that (1) under certain situations the optimal cluster size can be independent of the network dimensions, (2) the energy consumption of the transmitter's circuitry has no impact on the optimal cluster size, and (3) the energy consumption of the receiver's circuitry can substantially change the optimal number of clusters and more importantly, it can make a decision on whether or not it is worth to cluster the network.

The subsequent sections of this paper are organized as follows: Section 2 summarizes related work in this area while Section 3 introduces preliminary notions regarding the underlying network structure. Also, the architectures of LEACH, LEACH-Coverage, and DBS protocols are briefly reviewed. Section 4 presents the analysis for the energy-efficient cluster size. Sections 5 and 6 cover the simulation results and corresponding observations, respectively. Finally, Section 7 concludes this study.

2. Related work

During the last few years, the optimization of communication energy in wireless networks and particularly, the in wireless sensor networks has been investigated quite extensively. This optimization problem can be in the form of efficient number of hops in multi-hop networks [16], efficient cluster size (or number of clusters) in dense many-to-one sensor networks [17], and optimal transmission scheme in networks with different channel fading models (non-cooperative routing solutions investigated by Paschalidis et al. [18] and cooperative routing solutions with multiple relays studied by Madan et al. [19]). Furthermore, many clustering algorithms have been proposed as effective ways to organize communication and data processing in a sensor network (e.g., the guidelines provided by Olariu and Stojmenovic [20] to balance the energy load).

Authors in [17] derived an analytical model to achieve the optimal number of clusters for multi-hop wireless sensor networks. Their proposed analytical model is based on the assumption that all the sensor nodes employ multi-hop scheme to convey their data to cluster heads. In order to estimate the energy consumption for communications, they have considered an average hop count between a normal node and its nearest cluster head for the entire network without assuming a specific shape for the sensing field. In a recent work, Wang et al. [21] proposed a physical/medium access control/network cross-layer analytical design approach to determine the optimal number of clusters in dense sensor networks. They argue that different layers impose contradicting requirements on the cluster size.

In [6], the designers of LEACH communication protocol calculate the efficient number of clusters, whereby the network will save a huge amount of energy. However, their results solely specify an interval to which the optimal number of clusters belongs. It can be shown that in many network configurations, their analytical results produce a long interval and therefore, one has to find the optimal number of clusters through simulations for all the numbers that belong to the aforementioned interval. In the same manner, investigators in [14,15] do not provide the closed-form results.

Recent work by Kumar et al. [12] aims to extract the optimal number of clusters that would lead to minimum energy consumption. Although, their analytical approach is consistent with the results reported in [6], they make invalid assumptions about the average distance of the cluster head nodes to the BS. Likewise, the approaches presented in [11,14] attempt to solve the problem of optimal number of clusters, however, they do not take into account the energy consumed by the cluster head nodes when they receive the data from their cluster members.

Given that the focus in this work is on randomly distributed sensor networks where the nodes do not benefit from centralized coordination, some of the most closely-related approaches are mentioned. It should be noted that the notions of cluster size and number of clusters can be used interchangeably. This is because, for the protocols investigated in this paper, the cluster size is equal to total number of the nodes divided by the number of clusters.

Unlike approaches in the above-cited papers, a general framework to determine the optimal clustering parameters is presented in this paper. This way, several cluster-based protocols that are designed for sensor networks with many-to-one traffic patterns can become more energy-efficient through employing the appropriate number of clusters. DBS and LEACH-Coverage are two examples of such protocols; DBS protocol [8] provides a parallel version of LEACH algorithm to eliminate the energy imbalance that LEACH usually incurs; LEACH-Coverage protocol proposed in [7], is yet another extension to LEACH aiming at improving the sensing coverage offered by LEACH.

Although it has been shown that, in the sense of total energy consumption, the optimal number of clusters for all of the mentioned protocols is the same, none of them properly makes use of the optimal cluster size.

Regarding the cluster number variability, this problem has been addressed in [22,23]. However, it has not been investigated from the energy-efficiency point of view. It is shown that cluster number variability problem causes the energy consumption of the network not to be stable.

3. Preliminaries

The network model used in this paper and some considerations for developing an efficient cluster-based protocol are described in this section. In addition, the architectures of the LEACH, LEACH-Coverage, and DBS protocols are explained.

3.1. Network assumptions

In this work, for a sensor network the following assumptions have been made [8]:

(1) All sensor nodes are identical and are stationary after deployment. Each node is assigned a unique ID.
(2) Nodes are all time synchronized and have power control ability.
(3) The propagation channels are symmetric, i.e., two nodes can communicate using the same transmission power. As a matter of fact, real characteristics of low-power wireless links greatly differ from ideal ones; e.g., not every link can be converted to a symmetric link even at the maximum transmission power level [51].
(4) Nodes are assumed always have data to send and neighboring nodes have correlated data.
(5) Nodes are not equipped with GPS unit and therefore, they are not location-aware. However, they can approximate their distance from the BS based on the received signal strength (RSS) from the BS. It is noteworthy that for RSS values very close to sensitivity threshold of the receiver, local factors such as noise play the dominant role; thus, wireless communication researchers use a cutoff point for RSS values, below which the correlation between the RSS and the distance becomes questionable [52].

It should be noted that all the above assumptions are widely acceptable among previous researches [6–9].

The location-unawareness of the sensor nodes forces the clustering protocols to operate in a decentralized form which is our intention in this paper. Accordingly, the investigated protocols (LEACH, LEACH-Coverage, and DBS) are all location-unaware cluster-based protocols. Nevertheless, sensor nodes are able to estimate their approximate distance to the BS (see [8] for more details).

3.2. Radio energy dissipation model

The same energy model as the one introduced in [9,24,25] is used throughout the paper. In this energy model, E^{Tx}_{elec} and E^{Rx}_{elec} are defined as the energy being dissipated to run the transmitter's or receiver's circuitry, respectively, to send or receive one bit of the data packet. ε_{amp} represents the energy dissipation of the transmission amplifier to convey one bit of the data packet to the receiver node with a distance of $d = 1$ m away. As such, transmit (E_{Tx}) and receive (E_{Rx}) energies are calculated as follows:

$$\begin{aligned} E_{Tx}(l,d) &= lE^{Tx}_{elec} + l\varepsilon_{amp}d^n, \\ E_{Rx}(l,d) &= lE^{Rx}_{elec}, \end{aligned} \tag{1}$$

where l is the length of the transmitted/received message in bits, d represents the distance over which the data is communicated and n is the path loss exponent which is specified by radio propagation model. As it can be seen, the transmitter expends energy to run the radio electronics and power amplifier, while the receiver only expends energy to run the radio electronics.

In this paper, we consider both free space ($n = 2$, $\varepsilon_{amp} = \varepsilon_{fs}$) and two-ray multipath ($n = 4$, $\varepsilon_{amp} = \varepsilon_{mp}$) models to approximate signal attenuation as a function of the distance between transmitters and receivers. The former assumes exactly one path, which must be clear from obstacles, between the transmitter and the receiver. The latter takes into account one dominant reflection from the ground in addition to the clear path. However, the two-ray multipath model is highly optimistic for cases, where the transmitter–receiver separation distance is small. Therefore, in most applications, the two models are combined; the free space model is used at small distances, while the two-ray multipath model is used at larger distances. Both free space and two-ray models predict the signal strength as a deterministic function of distance and consequently, represent communication radius as an ideal circle.

Even though in the studied protocols the trend is to mostly keep the radio off, the receiver's radio must inevitably be kept on in anticipation of an incoming packet. Hence, a more accurate energy model should take the energy spent in idle listening period into account. Typically, the energy dissipated in the idle mode is almost equal to the energy consumption in the receive mode. Furthermore, the protocols investigated in this paper assume that the energy spent on sending control messages is significantly lower than that for data messages [6–8].

We acknowledge that the assumption of negligible energy consumption pertaining to idle listening and control messages do not hold in real deployments of low data rate sensor networks, let alone high data rate ones. Nevertheless, our corresponding solutions still lead to a worthwhile direction for future research.

3.3. Data aggregation

In cluster-based schemes, the cluster heads are responsible for aggregating their cluster members' data signals to produce a single representative signal, expending lE_{DA} nJ/signal for each l-bit input signal, where E_{DA} indicates the prorated cost of aggregation for a single bit.

Many authors in the literature assume that cluster heads have the ability to perfectly aggregate multiple incoming packets into one outgoing packet. Likewise, we do not deal with a particular data aggregation algorithm, but only with the amount of data generated in the aggregation process. In this paper, it is assumed that all sensors in a cluster sense the same event. This assumption will be met if either of two conditions holds: (1) the distance between nodes within a cluster is small with respect to the distance from which events can be sensed; (2) if the distance between events happening in the environment is large [8]. However, it is worth noting that aside from the data aggregation algorithm, other factors such as application requirements and the type of sensor data impact the aggregated data produced by the cluster head nodes [29].

3.4. Sensing coverage

The network sensing coverage is denoted by C and lies between 0 and 1. It is defined as the ratio of the network coverage area to the entire desired area that has to be sensed [7,8,30]. Therefore, for a randomly distributed sensor network with large number of sensor nodes, the network coverage is equal to 1 in the beginning of network operation. Presumably the sensing area within which a sensor is able to perform reliable sensing, is a circular area with radius of R_{sense}, and hence, the available sensing area of each node is πR^2_{sense}. It is obvious that the sensing areas of different sensors might overlap. Correspondingly, some protocols incorporate this common area into their clustering policies [7,30].

Please note that even if we precisely calibrate the sensors, still environmental impacts (e.g., obstacles) can severely change the sensing characteristics, causing irregular and non-uniform sensing patterns at different sensor nodes. Thus, assuming a circular sensing region for sensor nodes is an idealized assumption. Nonetheless, for a number of sensors such as ultraviolet sensors and inductive proximity sensors, the sensing region is indeed a circle.

3.5. Decentralized cluster-based protocols

The energy/lifetime optimization proposed in this paper is valid for several decentralized, self-organizing, and adaptive cluster-based protocols such as LEACH, LEACH-Coverage, and DBS. The communications and timing sequence of these protocols are nearly the same and the main difference arises from the cluster formation policies. All the aforementioned protocols make use of probabilistic approaches to distribute the energy load evenly among the sensor nodes. To this end, they utilize randomization to ensure that the cluster head role is shared equally among all the nodes.

These decentralized cluster-based protocols divide the schedule of the network into multiple rounds of fixed duration. As Fig. 1 suggests, each round consists of a set-up phase and a number of time frames that construct the steady-state phase. During the set-up phase some sensor nodes elect themselves as cluster heads by using a distributed algorithm (which is an exclusive characteristic of each protocol) performed in each node. Afterwards, the elected nodes announce their election as cluster head to the rest of the nodes in the network, and then other nodes organize themselves into local clusters by choosing the most appropriate cluster head (normally the closest cluster head). During the steady-state phase, within each frame the cluster heads receive sensor data from cluster members (according to TDMA schedule that was created and transmitted to them), and transfer the aggregated data to the BS. The transceiver of each non-cluster head can be turned off until the node's allocated transmission time (see Fig. 1). It should be noted that the cluster heads exploit data-aggregation to filter and compress the redundant data before the final transmission to the BS.

To obtain a thorough understanding of the studied decentralized protocols, Table 1 summarizes the main characteristics of them. As it can be seen, these protocols are all fully-distributed and they terminate in constant number of iterations. In addition, they incur low message overhead.

All of the aforementioned protocols have a mechanism to remember the nodes that have been elected as cluster head in recent rounds and this means that they consider the frequency of being a cluster head as one of their cluster formation metrics. However, LEACH-Coverage and DBS modify their cluster selection policies by giving more consideration to sensing coverage and the distance between the node and the BS, respectively. It should be noticed that none of these protocols takes into account the residual energy of the nodes. Taking the residual energy into consideration, one can avoid the formation of dead-areas [8] within the sensing field which is known as the energy hole (hot spot) problem [28,31,20]. However, the cluster-based protocol will no longer be decentralized (fully-distributed), since solving the energy hole problem requires global knowledge regarding the network in a periodic fashion and therefore, it implies the existence of a powerful node as a centralized controller.

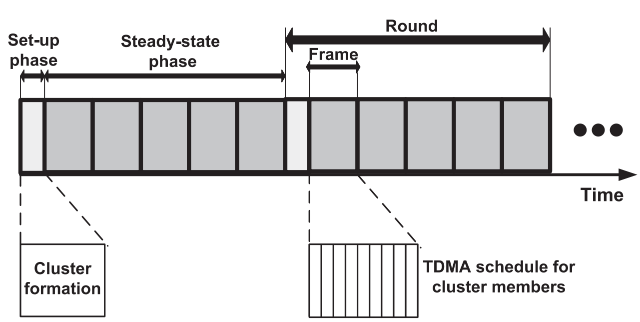

Fig. 1. Timing in the studied protocols.

Table 1
Comparison of decentralized cluster-based protocols.

Protocol	Cluster head selection metric			Main parameters
	Frequency	Sensing coverage	Distance to BS	
LEACH	Yes	No	No	k_{opt}
LEACH-Coverage	Yes	Yes	No	k_{opt}, η, α
DBS	Yes	No	Yes	k_{opt}, δp

Olariu and Stojmenovic [31] prove that regardless of the exploited routing scheme, the energy hole problem is unavoidable under certain conditions. Results reported in [8] verify that among the intended protocols, DBS is able to partly decrease the incidence of dead-areas.

Adaptive and dynamic cluster head election was first utilized in LEACH protocol [6] to guarantee the rotation of the energy intensive tasks (i.e., being cluster head) among all the nodes across the network. In LEACH, the optimal percentage of cluster head nodes (p) is equal to the ratio of the optimal number of clusters (k_{opt}) to the total number of sensor nodes in the network (N), i.e., $p = k_{opt}/N$. Correspondingly, the optimal cluster size would be equal to N/k_{opt}. During the cluster formation period, LEACH treats all the eligible sensor nodes without discrimination.

LEACH-Coverage scheme [7] aims at maintaining the network sensing coverage. To do so, different probabilities of being cluster head are applied based on effective sensing area of a node that is a function of the node density around a node. Thus, cluster heads will be usually selected in the high density areas where the death of few nodes does not affect the coverage of the network. LEACH-Coverage defines a parameter called estimated normalized effective sensing area ($0 < \eta(m) \leqslant 1$) for each node m in the sensor field. A node with a larger value of $\eta(m)$ will be assigned a smaller probability of being cluster head, and vice versa. It is evident that the nodes that die out first are the ones with a smaller normalized effective sensing area and therefore, their death has a minimal impact on the network sensing coverage, in that several nodes can compensate for their absence. The crucial nodes ($\eta = 1$) will not be elected as cluster heads, as no other node in the network can compensate for them. In other words, they are elected as cluster heads in every ∞ rounds. As it can be inferred, although the cluster head task is circulated among all the nodes, this circulation is weighted towards the nodes in high density areas. The investigators in [7] argue that the optimal number of clusters that yields the most energy-efficient behavior of the network is equal to that for LEACH protocol (k_{opt}). To adjust the number of clusters, LEACH-Coverage takes advantage of another parameter, denoted by α, which tunes the average number of clusters that are required in the network. It has been shown that for a dense sensor network, LEACH and LEACH-Coverage yield similar outcomes in terms of energy-efficiency [7,8]. However, LEACH-Coverage outperforms LEACH with regard to the sensing coverage.

As for DBS [8], the main idea is that the nodes with more distance from the BS should be cluster head less often than the nodes with closer distance to BS, to ensure that a great difference between energy levels of a near node and a far node would not occur. By applying this idea the system's energy-efficiency and network sensing coverage will be enhanced. To this end, DBS divides the entire sensing field into multiple segments with equal areas. It is straightforward to show that in each round of DBS the expected number of clusters (cluster heads) is the same as LEACH by noting that the areas of all segments are the same and the probabilities are distributed fairly around p (i.e., $p \pm \delta p, p \pm 2\delta p, \ldots$). Therefore, DBS does not affect the quality of data, since both protocols deliver the same amount of data per unit time. Hence, as with LEACH-Coverage, the optimal number of clusters for DBS should be equal to that for LEACH. It can be shown that LEACH can be thought of as a special case of DBS if one sets $\delta p = 0$.

Detailed description of LEACH, LEACH-Coverage, and DBS protocols, can be found in [6–8], respectively. In these three protocols, sensor nodes convey their data to the base station through two hops; there is no inter-cluster communication, and all cluster head nodes directly send their data to the base station. Consequently, cluster heads, whose distance from the BS is greater than their maximum reachable range, might be found in the network. Such connectivity issues can be alleviated by the emergence of new long-range tiny wireless nodes with line of sight transmission ranges of up to 1 km.

3.6. Lifetime definition

Lifetime of wireless sensor networks is the time span from the deployment to the instant when the network is considered nonfunctional. However, when a network should be considered nonfunctional is application-specific. It can be, for example, the instant when the first node dies, a certain percentage of sensors die, the network partitions, or loss of coverage occurs. In some cases it is necessary that all nodes stay alive as long as possible, since network quality decreases considerably as soon as one node dies. In these scenarios it is important to know when the first node dies. On the other hand, in some cases, sensors can be placed in proximity to each other. Thus, adjacent sensors could record related or identical data. Hence, the loss of a single or few nodes does not automatically diminish the quality of service of the network. In this study, the latter is considered as the network lifetime. Accordingly, the network lifetime is considered as the total available energy (E_{total}) divided by total energy consumption during a round (E_{round}), i.e., E_{total}/E_{round}.

4. Optimal cluster size

As mentioned in Section 2, the concepts of cluster size and number of clusters are used interchangeably in this paper. It is because the cluster size is equal to total number of the nodes divided by the number of clusters (i.e., N/k_{opt}).

The optimal construction of clusters (which is equivalent to setting the optimal probability for a node to become a cluster head) is very influential with regard to total energy expenditure and thus, the lifetime of the sensor network. In [5], the authors showed that if the clusters are not created in an optimal way, the total energy expenditure of the network is increased exponentially either when the number of clusters constructed is greater than the optimal number of clusters or especially when the number of clusters is less. This implies that there exists an optimal number of nodes that should be cluster heads (k_{opt}). If the number of cluster heads is less than k_{opt}, a remarkable number of nodes have to transmit their data very far to reach the cluster head, causing the global energy expenditure in the system to be large. On the other hand, if there are more than k_{opt} cluster heads, there is less data aggregation being performed locally and an enormous number of transmissions are required to monitor the entire sensing field.

In sensor networks that adopt decentralized cluster-based communication protocols, cluster formation algorithm ensures that the expected number of clusters per round equals k_{opt}, which is a predetermined system parameter. In this section, using the computation and communication energy model presented in Section 3, the optimal value for k_{opt} is analytically determined. In order to generalize the results, sensor field is assumed to be an arbitrary shape with area A. In a similar manner, no requirements with respect to the location of the BS and the radio propagation model (free space, two-ray, etc.) are imposed. However, in order to extract a closed-form expression for k_{opt}, subsequently, a number of special cases are considered to obtain an explicit parametric formula. It is shown that the optimization problem leads to an integral whose solution yields the expected nth power of distance between the BS and the sensor field, where n is the path loss exponent. This integral can be evaluated in closed-form under certain network configurations. Moreover, the optimization problem requires us to compute the expected squared distance between cluster members and their cluster head [6,32].

4.1. General optimization problem

In order to extend the network lifetime, one should minimize total energy expenditure of the network and therefore, total energy consumption during a round (see Fig. 1), denoted by E_{round}.

Assume that the sensing field is of an arbitrary shape of area A over which N nodes are distributed randomly and uniformly [33]. If there are k_{opt} clusters, there will be on average N/k_{opt} nodes per cluster (including one cluster head and $(N/k_{opt}) - 1$ non-cluster head nodes). Each cluster head dissipates energy receiving signals from the nodes, aggregating the signals, and transmitting the aggregated data to the BS [6]. Hence, the energy dissipated in a cluster head node during a single round with the assumption that each round has one frame (see Fig. 1) is:

$$E_{CH} = \left(\frac{N}{k_{opt}} - 1\right) lE^{Rx}_{elec} + \frac{N}{k_{opt}} lE_{DA} + lE^{Tx}_{elec} + l\varepsilon_{amp} d^{n}_{toBS}, \tag{2}$$

where l is the number of bits in each data message and d_{toBS} is the distance between the cluster head node and the BS. It should be noticed that by adopting perfect data aggregation, each cluster head needs to process N/k_{opt} signals of length l. Please note that the current analysis does not apply to cluster-based communication protocols using multi-hop approaches to convey cluster heads' data to the base station. We defer the multi-hop analysis to the Appendix.

On the other hand, each non-cluster head node transmits its data to the cluster head once per round. Understandably, the distance to the cluster head is small relative to the distance to the BS. Accordingly, the energy dissipation for non-cluster head nodes favorably follows the free-space model [34] and can be written as:

$$E_{non\text{-}CH} = lE^{Tx}_{elec} + l\varepsilon_{fs} d^{2}_{toCH}, \tag{3}$$

where d_{toCH} is the distance between the non-cluster head node and its cluster head. The area of each cluster is approximately A/k_{opt}. As stated in [6], one may argue that each cluster can be approximated by a circle of radius $(A/\pi k_{opt})^{1/2}$, where the cluster head resides at the center.

Since the expected squared distance between a random point in a circle of radius S and its center is $S^2/2$, the value of d^2_{toCH} can be determined with respect to A and k_{opt}. This can be obtained straightforwardly by drawing a hypothetical circle around the center (cluster head) and equating the area of this circle to half of the area of the original circle (area of the cluster). Solving the resulting equation for squared radius of the hypothetical circle, which is in fact equal to d^2_{toCH}, yields:

$$E[d^{2}_{toCH}] = \frac{A}{2\pi k_{opt}}. \tag{4}$$

By substituting (4) into (3), the energy used in each non-cluster head node per round is given by:

$$E_{non\text{-}CH} = lE^{Tx}_{elec} + \frac{l\varepsilon_{fs} A}{2\pi k_{opt}}. \tag{5}$$

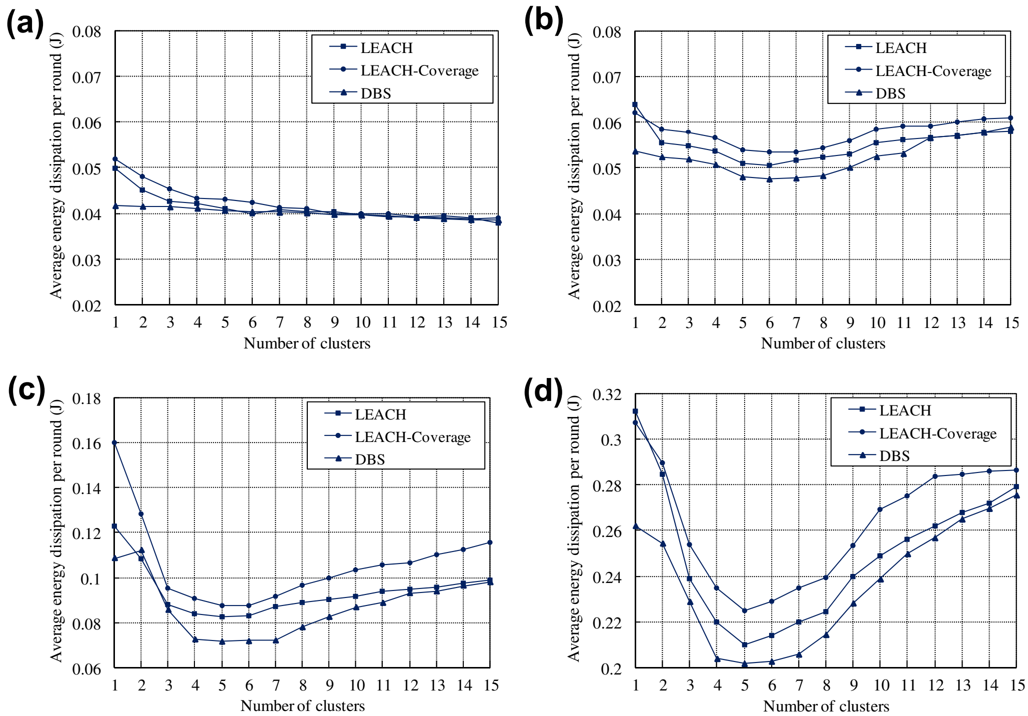

Fig. 2. Average energy spent per round when k_{opt} is varied between 1 and 15 in LEACH, LEACH-Coverage, and DBS (a) $M = 20$. (b) $M = 50$. (c) $M = 100$. (d) $M = 200$. These graphs show that the optimal number of clusters can be independent of the field dimensions under certain conditions, as predicted by Section 4.

The energy dissipated in an entire cluster during a single round can be expressed as follows:

$$E_{cluster} = E_{CH} + \left(\frac{N}{k_{opt}} - 1\right) E_{non\text{-}CH}. \tag{6}$$

Hence, E_{round} can be determined as follows:

$$\begin{aligned} E_{round} &= k_{opt} E_{cluster} \\ &= NlE^{Rx}_{elec} - k_{opt} lE^{Rx}_{elec} + NlE_{DA} + k_{opt} l\varepsilon_{amp} d^{n}_{toBS} + NlE^{Tx}_{elec} \\ &\quad + \frac{Nl\varepsilon_{fs}A}{2\pi k_{opt}} - \frac{l\varepsilon_{fs}A}{2\pi}. \end{aligned} \tag{7}$$

The first and second derivatives of E_{round} with respect to k_{opt} are:

$$\begin{aligned} &\frac{\partial E_{round}}{\partial k_{opt}} = l\varepsilon_{amp} d^{n}_{toBS} - lE^{Rx}_{elec} - \frac{Nl\varepsilon_{fs}A}{2\pi k^{2}_{opt}}, \\ &\frac{\partial^2 E_{round}}{k^{2}_{opt}} = \frac{Nl\varepsilon_{fs}A}{\pi k^{3}_{opt}} > 0. \end{aligned} \tag{8}$$

As (8) suggests, the non-negativity of the second derivative is evident and hence, by setting the first derivative to zero, the optimal number of clusters can be found:

$$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}A}{2\pi\left(\varepsilon_{amp} d^{n}_{toBS} - E^{Rx}_{elec}\right)}}. \tag{9}$$

The above equation for the optimal number of clusters points out that k_{opt} is principally determined by N, A, and d_{toBS}. Further, path loss exponent (n) and the energy consumption of the receiver's circuitry (E^{Rx}_{elec}) are also of importance as factors that can substantially change the optimal number of clusters and more fundamentally, they can make a decision on whether or not it is worth to perform clustering. To find the optimal value of k_{opt}, one has to substitute the minimum and maximum values of d_{toBS} in (9) and afterwards, the upper and lower bounds of k_{opt} can be achieved. Finally, k_{opt} will be selected from this interval according to simulations regarding the total energy expenditure of the network.

As such, it is desirable to replace d_{toBS} in (9) with a term containing other parameters such as N and A. However, unless the network configuration is symmetric in the sense of the shape of sensing field and the position of the BS, it is not feasible to obtain such a convenient term.

4.2. Case studies

On account of the facts mentioned in the previous section, a number of widely-used network configurations are considered for which a closed-form expression for the optimal number of clusters can be achieved. The optimal cluster size can then be simply derived, given that it equals N/k_{opt}. In such networks, the shape of the sensing field and/or the position of the BS with respect to the sensing field are, to some extent, symmetric. By symmetry considerations, the expected value of different powers of distance between the nodes across the sensing field and the BS (i.e., d_{toBS}, d^{2}_{toBS}, d^{4}_{toBS}) has been derived. Please find the derivations (A through P) at the end of this subsection.

Plugging the proper expected values into (9) yields the optimal number of clusters and thus, energy-efficient cluster size that the network should possess. Table 2 lists closed-form expressions to achieve the optimal number of clusters for different criteria.

Although the sensing field can be of any shape, circular (radius = R) and square-shaped (side length = M) sensing fields are of special interest in most of the research projects [9,13,31,32] due to their central and axial symmetry. Therefore, these two symmetric shapes are assumed for the sensing field to evaluate the optimal cluster size. Moreover, two radio propagation models (free space and two-ray) are followed for communications during the steady-state phase (see Section 3). As for location of the BS, to cover all possible configurations, the BS is put inside, outside and on the boundary of the sensing field. It should be noted that, in

Table 2
Closed-form expressions for the optimal number of clusters.?

Sensing field	Radio model	Location of BS	Optimal number of clusters	Optimal number of clusters (small E_{Rx})
Square ($M \times M$)	Free space	Center	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}M^2}{2\pi(\varepsilon_{fs}M^2/6 - E_{elec}^{Rx})}}$ (10)	$k_{opt} = \sqrt{\frac{3N}{\pi}} = 0.977\sqrt{N}$ (11)
		Corner	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}M^2}{2\pi(\varepsilon_{fs}2M^2/3 - E_{elec}^{Rx})}}$ (12)	$k_{opt} = \sqrt{\frac{3N}{4\pi}} = 0.489\sqrt{N}$ (13)
		Side's midpoint	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}M^2}{2\pi(\varepsilon_{fs}5M^2/12 - E_{elec}^{Rx})}}$ (14)	$k_{opt} = \sqrt{\frac{6N}{5\pi}} = 0.618\sqrt{N}$ (15)
		Outside (on the axis of symmetry)	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}M^2}{2\pi(\varepsilon_{fs}(M^2/6 + L^2) - E_{elec}^{Rx})}}$ (16)	$k_{opt} = \sqrt{\frac{NM^2}{2\pi(M^2/6 + L^2)}}$ (17)
	Two-ray	Center	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}M^2}{2\pi(\varepsilon_{mp}7M^4/180 - E_{elec}^{Rx})}}$ (18)	$k_{opt} = \sqrt{\frac{90N\varepsilon_{fs}}{7\pi\varepsilon_{mp}M^2}}$ (19)
		Corner	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}M^2}{2\pi(\varepsilon_{mp}28M^4/45 - E_{elec}^{Rx})}}$ (20)	$k_{opt} = \sqrt{\frac{45N\varepsilon_{fs}}{56\pi\varepsilon_{mp}M^2}}$ (21)
		Side's midpoint	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}M^2}{2\pi(\varepsilon_{mp}193M^4/720 - E_{elec}^{Rx})}}$ (22)	$k_{opt} = \sqrt{\frac{360N\varepsilon_{fs}}{193\pi\varepsilon_{mp}M^2}}$ (23)
		Outside (on the axis of symmetry)	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}M^2}{2\pi(\varepsilon_{mp}\left(\frac{7M^4}{180} + \frac{2}{3}M^2L^2 + L^4\right) - E_{elec}^{Rx})}}$ (24)	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}M^2}{2\pi\varepsilon_{mp}\left(\frac{7M^4}{180} + \frac{2}{3}M^2L^2 + L^4\right)}}$ (25)
Circle (radius = R)	Free space	Center	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}R^2}{R^2\varepsilon_{fs} - 2E_{elec}^{Rx}}}$ (26)	$k_{opt} = \sqrt{N}$ (27)
		Circumference	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}R^2}{\varepsilon_{fs}3R^2 - 2E_{elec}^{Rx}}}$ (28)	$k_{opt} = \sqrt{N/3} = 0.577\sqrt{N}$ (29)
		Outside	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}R^2}{2(\varepsilon_{fs}(R^2/2 + L^2) - E_{elec}^{Rx})}}$ (30)	$k_{opt} = \sqrt{\frac{NR^2}{2(R^2/2 + L^2)}}$ (31)
	Two-ray	Center	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}R^2}{2(\varepsilon_{mp}R^4/3 - E_{elec}^{Rx})}}$ (32)	$k_{opt} = \sqrt{\frac{3N\varepsilon_{fs}}{2\varepsilon_{mp}R^2}}$ (33)
		Circumference	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}R^2}{2(\varepsilon_{mp}10R^4/3 - E_{elec}^{Rx})}}$ (34)	$k_{opt} = \sqrt{\frac{3N\varepsilon_{fs}}{20\varepsilon_{mp}R^2}}$ (35)
		Outside	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}R^2}{2(\varepsilon_{mp}\left(\frac{R^4}{3} + L^4 + 2R^2L^2\right) - E_{elec}^{Rx})}}$ (36)	$k_{opt} = \sqrt{\frac{N\varepsilon_{fs}R^2}{2\varepsilon_{mp}\left(\frac{R^4}{3} + L^4 + 2R^2L^2\right)}}$ (37)

network configurations where the BS is outside the sensing field, the distance of the BS to the center of the sensing field is symbolized by L.

Furthermore, two main categories are considered and reported; networks containing sensor nodes with negligible receive energy (E_{Rx}) comparing to the transmit energy (E_{Tx}) (see, e.g., [1,2,28]) and networks in which the receive energy is comparable to the transmit energy (see, e.g., [24,35]).

Generally, the optimal number of clusters (k_{opt}) is determined by parameters such as total number of nodes (N), dimensions of the sensing field (M or R), distance of the nodes to the BS (d_{toBS}), energy consumption of the receiver's circuitry (E_{elec}^{Rx}), and the energy dissipation of the transmitter amplifier (ε_{amp}), i.e.,

$$k_{opt} = f\left(N, M \text{ or } R, d_{toBS}, E_{elec}^{Rx}, \varepsilon_{amp}\right). \tag{38}$$

It can be inferred from (9) and Table 2 that the optimal number of clusters and hence, the optimal cluster size are always independent of energy consumption of the transmitter's circuitry (E_{elec}^{Tx}). This can be justified through the fact that E_{elec}^{Tx} does not appear in the first derivative of E_{round} (refer to Section 4.1).

Yet another interesting property pointed out by Table 2 is that under certain conditions, the optimal cluster size is independent of the size of the sensing field. Such conditions imply that E_{elec}^{Rx} should be small comparing to E_{elec}^{Tx}, the wireless transmissions are governed by free space radio propagation model, and the BS is not located outside of the sensing field. As Table 2 suggests, if the above requirements are met, the optimal number of clusters and thus, the optimal cluster size will solely depend on the number of sensor nodes across the network, that is $k_{opt} = f(N)$. More specifically, if the sensing area is square-shaped or circular, the optimal number of clusters an be expressed as $B\sqrt{N}$, where B is a constant with maximum value of 1. Simulation results in Section 5 will verify these claims.

The optimal number of clusters has its largest value when the BS is located at the center of the sensing field (for instance, compare (2) against (2) and (2)). This is as anticipated, since the expected value of the nth power of distance between the cluster heads and the BS ($E[d^n_{toBS}]$) has its least value in such configuration. Correspondingly, as the BS moves from the center of the sensing area towards the boundary and then to the outside of the sensing field, optimal number of clusters will be reduced.

Eventually, in case the BS is located very far from the sensing field, the network will have one cluster whose size and area are equal to N and A, respectively. In other words, only one long-haul transmission is done within each round to save energy. Likewise, whenever E^{Rx}_{elec} dominates $\varepsilon_{amp}d^n_{toBS}$ (transmit amplifier energy), according to (9), k_{opt} becomes very large so that it is no longer worth to cluster the sensing field and thus, having N clusters of size 1 is more efficient in terms of total energy expenditure of the network. In general, it should be noticed that whenever the value of each expression reported in Table 2 becomes undefined, one can draw the conclusion that the optimal number of clusters might be equal to N, that is, all the nodes acting as cluster head and there is no intra-cluster communication.

As previously mentioned, the expected value of different powers of distance between the nodes across the sensing field and the BS (i.e., $d_{toBS},\ d^2_{toBS},\ d^4_{toBS}$) has been derived as follows:

A. Calculation of d_{toBS} (BS at the center of the square)

Let the sensing field be in the shape of a square of side M and assume that the BS is located at the center of the sensing field. The probability P that the distance between a randomly chosen point and the BS located at the center of the square is less than x should be obtained. Calculation of P for cases in which x is less than or equal to $M/2$ is trivial. However, as Fig. 3 suggests, this calculation is more complicated in cases where the hypothetical circle around the center of the square intersects with the perimeter of the square.

The value of angle φ in Fig. 3 is used to derive the area of the portion of square that overlaps with the hypothetical circle (see the gray area). According to Fig. 3, the angle φ is given by:

$$\varphi = \arctan\left(\frac{\sqrt{x^2 - M^2/4}}{M/2}\right). \tag{39}$$

It is evident that the minimum and maximum value of φ are 0 rad and $\pi/4$ rad, respectively. Bases on the above discussions, P can be determined as follows:

$$P(d_{toBS} \leqslant x) = \begin{cases} \frac{\pi x^2}{M^2} & \text{if } 0 \leqslant x \leqslant \frac{M}{2}, \\ \frac{\pi x^2 - 4[x^2\varphi - L(M/2)]}{M^2}, & \text{if } \frac{M}{2} \leqslant x \leqslant \frac{\sqrt{2}M}{2}. \end{cases} \tag{40}$$

Correspondingly, the probability density function is given by:

$$f(x) = \begin{cases} \frac{\pi x}{M^2/2} & \text{if } 0 \leqslant x \leqslant \frac{M}{2}, \\ \frac{\pi x}{M^2/2} - \frac{2x}{M^2/4}\varphi & \text{if } \frac{M}{2} \leqslant x \leqslant \frac{\sqrt{2}M}{2}. \end{cases} \tag{41}$$

To find the expected value of the distance from the center to the entire area of the square, one must integrate $xf(x)$ in the interval $\left[0, M\sqrt{2}/2\right]$, which yields:

$$E[d_{toBS}] = \frac{M(\sqrt{2} + \ln(1+\sqrt{2}))}{6} \approx 0.383M. \tag{42}$$

B. Calculation of d^2_{toBS}(BS at the center of the square)

Let sensing field be in the shape of a square of side M and assume that the BS is located at the center of the sensing field. Further, assume that the origin of the coordinate system resides at the center of the field. The expected squared distance from the BS can be obtained as follows:

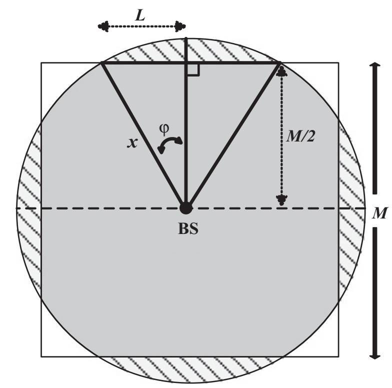

Fig. 3. Square-shaped sensing field with BS at the center and $x > M/2$.

$$E[d^2_{toBS}] = E[x^2 + y^2] = \frac{1}{M^2}\int_{-M/2}^{M/2}\int_{-M/2}^{M/2}(x^2+y^2)dx\,dy = \frac{4M^4}{24M^2} = \frac{M^2}{6}. \tag{43}$$

C. Calculation of d^4_{toBS} (BS at the center of the square)

The expected fourth power of distance from the BS is:

$$\begin{aligned} E[d^4_{toBS}] &= E[(x^2+y^2)^2] = E[x^4] + E[y^4] + 2E[x^2]E[y^2] \\ &= \frac{1}{M}\int_{-M/2}^{M/2} x^4 dx + \frac{1}{M}\int_{-M/2}^{M/2} y^4 dy + 2\frac{M^2}{12}\frac{M^2}{12} \\ &= \frac{M^4}{80} + \frac{M^4}{80} + \frac{M^4}{72} = \frac{7M^4}{180}. \end{aligned} \tag{44}$$

D. Calculation of d_{toBS}(BS at the center of the circle)

For a circular sensing field, the expected distance from the BS (located at the center) to the entire area of the circle is:

$$E[d_{toBS}] = \frac{1}{A}\int_{r=0}^{r=R} rf(r)dr = \frac{1}{\pi R^2}\int_{r=0}^{r=R} r \cdot 2\pi r dr = \frac{2R}{3}. \tag{45}$$

E. Calculation of d^2_{toBS}(BS at the center of the circle)

Similarly, the expected squared distance between the BS to the circular sensing field can be expressed as:

$$E[d^2_{toBS}] = \int_{\theta=0}^{2\pi}\int_{r=0}^{r=R} r^2\rho(r,\theta) r dr\, d\theta = \frac{1}{\pi R^2}\int_{\theta=0}^{2\pi}\int_{r=0}^{R} r^3 dr\, d\theta = \frac{R^2}{2}. \tag{46}$$

F. Calculation of d^4_{toBS}(BS at the center of the circle)

In the same manner, the expected fourth power of distance from the BS is given by:

$$E[d^4_{toBS}] = \int_{\theta=0}^{2\pi}\int_{r=0}^{r=R} r^4\rho(r,\theta) r dr\, d\theta = \frac{1}{\pi R^2}\int_{\theta=0}^{2\pi}\int_{r=0}^{R} r^5 dr\, d\theta = \frac{R^4}{3}. \tag{47}$$

G. Calculation of d_{toBS}(BS on perimeter of the square)

We calculate the expected distance from the points within a square-shaped sensing field to BS that resides in the corner of the field and more generally, a BS whose location is an arbitrary point on perimeter of the sensing field.

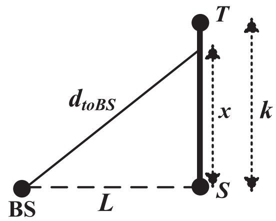

Fig. 4. Line segment TS represents the location of the sensor nodes and the BS is an external point.

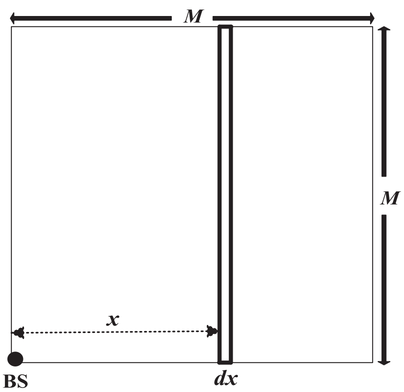

Fig. 5. Square-shaped sensing field with BS in the corner.

First consider a network where all of the nodes reside on a segment of a straight line (line segment TS in Fig. 4) of length k and the BS is located at distance L from TS.

One can verify that the expected distance from the BS to the line segment TS can be determined through the following expression:

$$E[d_{toBS}] = \frac{1}{2k}\left(k\sqrt{L^2+k^2} + L^2 \arcsin h\frac{k}{L}\right). \tag{48}$$

As illustrated in Fig. 5, suppose that the BS is in the corner of a square-shaped sensing field.

The contribution of the narrow band shown in Fig. 5 to the total distance from the BS is:

$$E[d_{toBS}] = \frac{1}{2}\left(M\sqrt{x^2+M^2} + x^2 \arcsin h\frac{M}{x}\right)dx. \tag{49}$$

Therefore, the expected distance $E[d_{toBS}]$ to all the points (sensor nodes) within the square is given by:

$$\begin{aligned}E[d_{toBS}] &= \frac{1}{2M^2}\int_0^M \left(M\sqrt{x^2+M^2} + x^2 \arcsin h\left(\frac{M}{x}\right)\right)dx \\ &= \frac{1}{2M^2}\left(\frac{M}{2}\left[x\sqrt{x^2+M^2} + M^2 \arcsin h\left(\frac{x}{M}\right)\right]_0^M \right.\\ &\quad \left. + \left[\frac{x^3}{3}\arcsin h\left(\frac{M}{x}\right) + \frac{Mx}{6}\sqrt{x^2+M^2} - \frac{M^3}{6}\arcsin h\left(\frac{x}{M}\right)\right]_0^M\right) \\ &= \frac{1}{6M^2}\left(M^3 \arcsin h(1) + M^3 \arcsin h(1) + 2\sqrt{2}M^3\right) \\ &= \frac{M}{3}\left(Ln\left(1+\sqrt{2}\right) + \sqrt{2}\right) \approx 0.765M.\end{aligned} \tag{50}$$

Hence, the general problem of the expected distance from the nodes in the interior of the square to the BS that is located on an arbitrary point on perimeter of the square (see Fig. 6) can be solved.

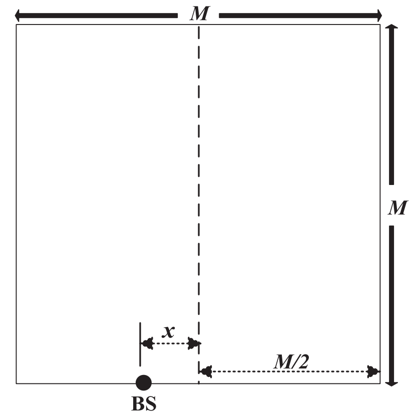

Fig. 6. Square-shaped sensing field with BS on perimeter.

Let the distance between the midpoint and BS be x. Then the expected distance between the interior nodes and the BS can be evaluated by two consecutive applications of the expression for corner:

$$\begin{aligned}E[d_{toBS}] = \frac{1}{6M^2}&\left(\left(\frac{M}{2}+x\right)^3 \arcsin h\left(\frac{2M}{M+2x}\right) + M^3 \arcsin h\left(\frac{M+2x}{2M}\right)\right.\\ &+ M(M+2x)\sqrt{\left(\frac{M}{2}+x\right)^2 + M^2} + \left(\frac{M}{2}-x\right)^3 \arcsin h\left(\frac{2M}{M-2x}\right) \\ &\left. + M^3 \arcsin h\left(\frac{M-2x}{2M}\right) + M(M-2x)\sqrt{\left(\frac{M}{2}-x\right)^2 + M^2}\right).\end{aligned} \tag{51}$$

In the similar manner, consider a network where the BS is put at the center of the side (on midpoint, i.e., $x = 0$ in see Fig. 6). Substituting zero in place of x yields:

$$\begin{aligned}E[d_{toBS}] &= \frac{1}{6M^2}\left(\left(\frac{M}{2}\right)^3 \arcsin h(2) + M^3 \arcsin h\left(\frac{1}{2}\right) + M^2\sqrt{\frac{5M^2}{4}}\right.\\ &\quad \left. + \left(\frac{M}{2}\right)^3 \arcsin h(2) + M^3 \arcsin h\left(\frac{1}{2}\right) + M^2\sqrt{\frac{5M^2}{4}}\right) \\ &= \frac{M}{3}\left(\frac{\arcsin h(2)}{8} + \arcsin h\left(\frac{1}{2}\right) + \frac{\sqrt{5}}{2}\right) = 0.593M.\end{aligned} \tag{52}$$

H. Calculation of d^2_{toBS} (BS on perimeter of the square)

Now suppose that the BS is located at the origin of the coordinate system which is considered to be in the bottom left corner of the sensing field. The expected squared distance from the BS can be obtained as follows:

$$\begin{aligned}E[d^2_{toBS}] &= E[x^2+y^2] = E[x^2] + E[y^2] \\ &= \frac{1}{M}\int_0^M x^2dx + \frac{1}{M}\int_0^M y^2dy = \frac{2M^2}{3}.\end{aligned} \tag{53}$$

As another configuration, let the BS be located on the midpoint of the bottom side of the sensing field, while the origin is still in the bottom left corner. This time, the expected squared distance from the BS is determined by:

$$\begin{aligned}E[d^2_{toBS}] &= E\left[\left(x-\frac{M}{2}\right)^2 + y^2\right] = E[x^2] - ME[x] + \frac{M^2}{4} + E[y^2] \\ &= \frac{1}{M}\int_0^M x^2dx - \frac{M}{M}\int_0^M xdx + \frac{M^2}{4} + \frac{1}{M}\int_0^M y^2dy = \frac{5M^2}{12}.\end{aligned} \tag{54}$$

I. Calculation of d^4_{toBS}(BS on perimeter of the square)

With regards to the expected fourth power of distance from the BS to the sensing field, suppose that the BS and origin of the coordinate are both located in the bottom corner of the square. The expected fourth power of distance can be expressed as follows:

$$E[d^4_{toBS}] = E[(x^2+y^2)^2] = E[x^4]+E[y^4]+2E[x^2]E[y^2] = \frac{1}{M}\int_0^M x^4dx + \frac{1}{M}\int_0^M y^4dy + 2\frac{M^2}{3}\frac{M^2}{3}\frac{M^4}{5}+\frac{M^4}{5}+\frac{2M^4}{9}=\frac{28M^4}{45}. \quad (55)$$

However, if the BS is put on the midpoint of the bottom side of the sensing field, we obtain:

$$\begin{aligned}E[d^4_{toBS}] &= E\left[\left(\left(x-\frac{M}{2}\right)^2+y^2\right)^2\right]\\ &= E[x^4]-2ME[x^3]+\frac{M^2}{2}E[x^2]+2E[x^2]E[y^2]+M^2E[x^2]\\ &\quad -\frac{M^3}{2}E[x]-2ME[x]E[y^2]+\frac{M^4}{16}+\frac{M^2}{2}E[y^2]+E[y^4]\\ &= \frac{M^4}{5}-2M\frac{M^3}{4}+\frac{M^2}{2}\frac{M^2}{3}+2\frac{M^2}{3}\frac{M^2}{3}+M^2\frac{M^2}{3}-\frac{M^3}{2}\frac{M}{2}\\ &\quad -2M\frac{M}{2}\frac{M^2}{3}+\frac{M^4}{16}+\frac{M^2}{2}\frac{M^2}{3}+\frac{M^4}{5}\\ &= \frac{2M^4}{5}-\frac{11M^4}{16}+\frac{5M^4}{9}=\frac{193M^4}{720}.\end{aligned} \quad (56)$$

J. Calculation of d_{toBS}(BS on circumference of the circle)

The expected distance from the BS to the entire area of the circular sensing field assuming that the BS resides on circumference of circle is given by:

$$E[d_{toBS}] = \frac{32R}{9\pi}. \quad (57)$$

K. Calculation of d^2_{toBS}(BS on circumference of the circle)

Likewise, the expected squared distance between the BS (residing on circumference) to the circular sensing field can be expressed as:

$$\begin{aligned}E[d^2_{toBS}] &= \int_{x=0}^{x=x_{\max}}\int_{y=0}^{y=y_{\max}}[x^2+(y+R)^2]\rho(x,y)dx\,dy\\ &= \frac{1}{A}\int_{\theta=0}^{2\pi}\int_{r=0}^{r=R}(r^2+2rR\cos\theta+R^2)rdrd\theta=\frac{3R^2}{2}.\end{aligned} \quad (58)$$

L. Calculation of d^4_{toBS}(BS on circumference of the circle)

In the same manner, the expected fourth power of distance from the sensing area to the BS which is positioned on the circumference of the sensing area can be written as follows:

$$\begin{aligned}E[d^4_{toBS}] &= \int_{x=0}^{x=x_{\max}}\int_{y=0}^{y=y_{\max}}[x^2+(y+R)^2]^2\rho(x,y)dx\,dy\\ &= \frac{1}{\pi R^2}\int_{\theta=0}^{2\pi}\int_{r=0}^{R}(r^2+2rR\cos\theta+R^2)^2rdr\,d\theta\\ &= \frac{1}{\pi R^2}\int_{\theta=0}^{2\pi}\left(\frac{7R^6}{6}+R^6\cos^2\theta\right)=\frac{10R^4}{3}.\end{aligned} \quad (59)$$

M. Calculation of d^2_{toBS} (BS outside of the square)

Assume that the BS is located outside the sensing field whose shape is a square of side M. For the sake of symmetry, the BS is positioned above the midpoint of the side in such a way that the distance from the BS to the center of the sensing field is L. Clearly, L is greater than $M/2$ (i.e., $L = M/2 + K$). The origin of the coordinate system is considered to be in the bottom left corner of the sensing field. In this case, the expected squared distance between the sensing field and the BS is:

$$\begin{aligned}E[d^2_{toBS}] &= E\left[\left(x-\frac{M}{2}\right)^2+\left(y-\frac{M}{2}-L\right)^2\right]\\ &= E[x^2]-E[Mx]+E\left[\frac{M^2}{4}\right]+E[y^2]-\left(\frac{M}{2}+L\right)E[y]+E\left[\left(\frac{M}{2}+L\right)^2\right]\\ &= \frac{M^2}{6}+L^2=\frac{M^2}{6}+(M/2+K)^2=\frac{5M^2}{12}+MK+K^2.\end{aligned} \quad (60)$$

N. Calculation of d^4_{toBS}(BS outside of the square)

As for the expected fourth power of distance from the BS to the sensing field, suppose that the BS is located outside a sensing field of square shape. More specifically, suppose that it L units away from the center of the square. The expected fourth power of distance will be:

$$\begin{aligned}E[d^4_{toBS}] &= E\left[\left(\left(x-\frac{M}{2}\right)^2+\left(y-\frac{M}{2}-L\right)^2\right)^2\right]=\cdots\\ &= \frac{7M^4}{180}+\frac{2M^2L^2}{3}+L^4.\end{aligned} \quad (61)$$

O. Calculation of d^2_{toBS}(BS outside of the circle)

The expected squared distance between BS that resides outside of a circle with radius R to the interior sensor nodes can be written as:

$$\begin{aligned}E[d^2_{toBS}] &= \int_{x=0}^{x=x_{\max}}\int_{y=0}^{y=y_{\max}}[x^2+(y+L)^2]\rho(x,y)dx\,dy\\ &= \frac{1}{A}\int_{\theta=0}^{2\pi}\int_{r=0}^{r=R}(r^2+2rL\cos\theta+L^2)rdr\,d\theta=\frac{R^2}{2}+L^2,\end{aligned} \quad (62)$$

where L is the distance between the center of the circle and the BS.

P. Calculation of d^4_{toBS}(BS outside of the circle)

In the same manner, the expected fourth power of distance from the circular sensing area to the BS which is positioned L units away from the center of the sensing area can be calculated as:

$$\begin{aligned}E[d^4_{toBS}] &= \int_{x=0}^{x=x_{\max}}\int_{y=0}^{y=y_{\max}}[x^2+(y+L)^2]^2\rho(x,y)dx\,dy\\ &= \frac{1}{\pi R^2}\int_{\theta=0}^{2\pi}\int_{r=0}^{R}r^2+2rL\cos\theta+L^2\Big)^2rdr\,d\theta\\ &= \frac{1}{\pi R^2}\int_{\theta=0}^{2\pi}\left(\frac{R^6}{6}+\frac{R^4L^2}{2}+R^4L^2\cos^2\theta+\frac{R^2L^4}{2}\right)\\ &= \frac{R^4}{3}+L^4+2R^2L^2.\end{aligned} \quad (63)$$

5. Simulation results

In this section the mathematical results presented in Section 4 are further validated through extensive simulations on randomly distributed sensor networks. In the simulations, three decentralized cluster-based protocols, namely LEACH, LEACH-Coverage, and DBS are considered as data gathering schemes. As simulation platform, a custom simulator written in MATLAB is used, whereby various parameters such as energy consumption, network lifetime, and sensing coverage can be evaluated for cluster-based communication protocols. Further, this simulator can simulate routing and clustering policies and provide statistical information such as the residual energy of each node, communication paths, wasteful transmissions (if any) during the network operation, and the energy waste due to variability of the number of clusters.

Closed-form expressions presented in Table 2 are evaluated over a set of sensor networks. For a single network configuration, one hundred sensing fields are generated over which the above-mentioned protocols are run to accomplish the average result. In each network, the sensor nodes are randomly deployed on a 2D sensing field. Table 3 lists all simulation parameters (including network model, energy model, and protocol-specific parameters) and their corresponding values. It should be noted that although in the simulations, only integer numbers are assigned to k_{opt}, using fractional numbers is also logical.

To investigate the effect of network dimensions on k_{opt}, sensing field is assumed to be square-shaped with side length ranging from 20 m to 200 m and the BS is put in the corner of the sensing field. Fig. 2 shows the average energy dissipation per round as a function of the number of clusters for sensing fields with different side lengths (M) in the above range.

For each sensing field, we varied the number of clusters between 1 and 15 and ran the studied protocols 100 times. Simulation time is chosen to be 500 rounds so that all three protocols operate stably, in that all of the nodes are alive in this time interval. It should be noted that the pattern at which the nodes die in the network (i.e., energy imbalance issues) is not the objective of the simulations. In this regard, more details can be found on [8].

Table 3
Parameters used in simulations.

Parameter	Value
Network model	
Field span	Square: 20 m × 20 m, 50 m × 50 m, 100 m × 100 m, 200 m × 200 m
Location of BS	Corner: (0,0)
N	100
Sensing radius (R_{sense})	2 m, 7.5 m, 10 m, 15 m
Maximum transmission distance	305 m
Packet size (l)	500 bytes
Number of frames per round	1
Energy model	
Initial energy of each sensor (E_0)	2 J
E_{elec}^{Tx}	50 nJ/bit
E_{elec}^{Rx}	50 nJ/bit
ε_{fs}	100 pJ/bit/m^2
Path loss exponent (n)	2
E_{DA}	5 nJ/bit/signal
LEACH	
k_{opt}	1–15
LEACH-Coverage	
k_{opt}	1–15
$\eta(m)$	0–1
$\alpha(m)$	$(k_{opt}/N)/\Phi(m)$ [7,8]
DBS	
k_{opt}	1–15
Number of segments	1–5
δp	0–0.1

In a more specific manner, in the first set of simulations, $M = 20$ m, 0 m < d_{toBS} < 28.28 m. Replacing the energy parameters in (9) with their corresponding values from Table 3, the optimal number of clusters k_{opt} is expected to reside in the interval [23.03, 100]. However, according to the associated closed-form expression (2) presented in Table 2, k_{opt} should be equal to 100, that is, if each cluster contains one node (no intra-cluster communication), the total energy consumption will be minimized.

From Fig. 2(a), it is evident that having more number of clusters results in lower average energy dissipation per round and it is why the trend of the function (average energy dissipation per round versus the number of clusters in Fig. 2(a)) is decreasing. According to simulations, the most energy-efficient outcomes are experienced when $k_{opt} = 100$. As it can be seen, the simulation results are in complete agreement with analytical results.

In the second set of simulations, sensing field dimensions are scaled, i.e., $M = 50$ m, 0 m < d_{toBS} < 70.71 m, while the number of nodes are kept intact, i.e., $N = 100$. Plugging the proper energy parameter values into (9), the optimal number of clusters k_{opt} should fall within the interval [5.05, 100]. Fig. 2(b) illustrates that applying six clusters across the sensing field yields the minimum amount of energy dissipation for all the decentralized protocols targeted in this study. This is in accordance with (2) which suggests that k_{opt} should be set to 5.84. In the same manner, two more sets of simulations have been performed on larger sensing fields, i.e., $M = 100$, 200 m. By the same analysis, k_{opt} is expected to reside respectively in intervals [2.86, 100] and [2.83, 100] for these two configurations. Fig. 2(c) and (d) depict the average energy dissipated per round against k_{opt} for the two recent configurations. It can be observed that the optimal number for clusters for both cases is around 5 which excellently agrees with mathematical calculations established upon (2) presented in Table 3.

In two recent configurations, owing to the larger field and power control capability of the sensor nodes, the energy consumption of the receiver electronics (E_{elec}^{Rx}) is dominated by power amplifier energy ($\varepsilon_{fs} d_{toBS}^2$). As such, the simplified closed-form expression (2) reported in Table 3 (in this case $k_{opt} = 0.489\sqrt{N}$) can be utilized with a very small error.

As mentioned in Section 4, (2) advises that the optimal number of clusters does not depend on the size of the network. Simulation results verify this property by showing that the optimal number of clusters is around 5, regardless of the network dimensions (i.e., $M = 50$, $M = 100$ and $M = 200$). At the first glance, this result seems abnormal; however, intuitively, one can think of a network of size 100 with 100 nodes as a network of size 200 with 100 nodes that has only been scaled by a factor of 2. Therefore, the number of cluster heads should not differ between these two networks. It also should be noted that E_{elec}^{Rx} is presumed to be very small. If E_{elec}^{Rx} is large, it will be in the form of a constant overhead that can fundamentally make the cluster-based schemes questionable.

In another attempt to further confirm the above-mentioned observation, the network size (i.e., the side length of the square-shaped field) is kept unchanged ($M = 100$) while the number of the nodes is increased to 200 nodes and 400 nodes for the same network. In like manner, protocols LEACH, LEACH-Coverage, and DBS are simulated. By averaging over one hundred simulation runs, it has been verified that the optimal number of clusters for these two new networks are respectively 7 and 10 which agrees well with (2). This verifies that under certain assumptions (refer to Section 4.2), the simplified closed-form expressions can promisingly provide the best value of k_{opt} in regard of energy-efficiency.

None of the investigated protocols guarantees the placement and/or number of cluster head nodes. In particular, in LEACH and LEACH-Coverage, there is a possibility for a round to have no cluster head at all. In this regard, Monte Carlo simulations [23] show that the probability that there is no cluster head (cluster) is high when k_{opt} is small. However, LEACH and LEACH-Coverage do not clearly specify the task of a sensor network in a round whose number of clusters is zero. Fortunately, DBS takes some action to always guarantee the existence of at least one cluster head to overcome this problem. Therefore, as all graphs in Fig. 2 indicate, using LEACH and LEACH coverage, once k_{opt} becomes close to one, an abrupt increase in the energy dissipation can be substantiated. This energy waste can be attributed to situations where there is no cluster head across the network and this problem exhibits itself as inutile time and energy consumption in LEACH and LEACH-Coverage.

In all of the studied protocols, k_{opt} solely determines the expected number of clusters that each protocol should possess to be energy-efficient throughout the network operation. In fact, the number of clusters is variable and this number is distributed in a range around k_{opt}. For instance, in a 100-node network, when $k_{opt} = 5$, the number of cluster heads varies from 0 to 34 and the percentage of rounds that exactly have five cluster heads is less than 20%. Also, we observed that within 100 rounds, there is on average one round whose number of cluster heads is zero. Obviously, as k_{opt} decreases, more occurrences of rounds with zero number of clusters are foreseen.

The energy waste due to variability of the number of clusters could be eliminated by rejecting certain low values for k_{opt} that lead to higher chances of having zero clusters (e.g., $k_{opt} < 5$ for a 100-node network) and choosing the closest value to the one that closed-form analysis recommends. In such situations, the closed-form expressions reported in Table 2 are no longer valid.

6. Discussions

As mentioned in Sections 4 and 5, energy consumption of the receiver's circuitry (E^{Rx}_{elec}) is of special importance as a factor that can substantially change the optimal number of clusters and more fundamentally, it can make a decision on whether or not it is worth to cluster the network. Throughout the simulations, it was assumed that the receive energy (E_{Rx}) is smaller than the transmit amplifier energy ($\varepsilon_{amp} d^n_{toBS}$) which can construct (e.g., in wireless LANs) a significant portion [36] of the maximum transmit energy (E_{Tx}). Thereby, as we increased the size of the sensing field, more improvements in terms of energy consumption were achieved. Most of the previous studies make similar assumptions regarding the transmit and receive costs [2,6,7,12,16,24,25,28,31]. For instance, investigators in [2] argue that the energy spent to receive the packets is negligible with respect to the energy used for the packet transmission. Correspondingly, they mention some cases for which this condition occurs [1]. Among the commercially-available motes, Xbee ZNet 2.5 Zigbee Modules from Digi International [37] conform to this condition, in that the transmit energy is up to 6.5 times the receive energy. Likewise, on Crossbow Mica2 motes [38], the maximum transmit energy is 2.5 times the receive energy [39,40].

On the other hand, Chang and Tassiulas [35] reason that the receiver's circuitry is in general more complex than its counterpart in the transmitter side. Therefore, it consumes more energy than the transmitter's circuitry within the same order of magnitude. One can verify that the receive energy (E_{Rx}) would be comparable to the transmit energy (E_{Tx}) by considering the energy consumption of the power amplifier (i.e., output transmitter antenna). Among the available motes in the market, on ANT motes from Dynastream Innovations [42], the receive energy is greater than the maximum transmit energy by a factor of 1.2. This is because the maximum transmission range is only 20 m and hence, the amplifier energy cannot grow. Similarly, on TI mote EZ430-RF2500 [43], transmit and receive energies are almost equal. In this context, Bhardwaj and Chandrakasan [36] explain that among various factors that determine how E_{Tx} and E_{Rx} compare, the regulatory limit on output amplifier (i.e., radiated power from the antenna) is the governing one. When a high output power is supported, the transmitter is likely to dominate due to energy dissipated by power amplifier. When regulatory limits are tight, as in ultra-wideband systems, the receiver, owing to its more complex signal conditioning and processing, dominates the transmit energy [44]. Further explanations regarding the wireless protocols and corresponding parameters (e.g., data rate) that each of the above motes exploits, are omitted due to space limitations.

Results in the previous section suggest that the three cluster-based protocols targeted in this work, behave similarly in terms of energy-based lifetime in a dense sensor network. It can be shown that for sparse sensor networks, they provide different results. Besides, previous papers [7,8] have shown that these protocols produce different outcomes regarding the sensing coverage. This is because both LEACH-Coverage and DBS attempt to distribute the energy load more uniformly across the network.

Generally, future amendments to our approach include but are not limited to:

- Considering partial data aggregation instead of perfect data aggregation. This way, the reliability of data within each cluster (as a quality of service) should be taken into account as well.
- Utilizing a more practical radio energy model [44,45] in place of the first-order radio energy model.
- Exploring the parameter space to a higher extent, e.g., conducting simulations for different transmit/receive energies and finding the corresponding limits.
- Addressing the density of the nodes in the network [46], i.e., sparse network versus dense network (closely-spaced nodes).
- Investigating other node distributions across the network [47], such as Gaussian distribution.
- Integrating packet size optimization considering the errors imposed by the environment [19].
- Performing optimization for energy balancing (coverage), and
- Exploiting multi-hop schemes for networks in which the BS is not accessible by all the nodes. This approach might yield more energy savings as well [16].

It is worth mentioning that the results provided in this paper are not limited to the studied protocols; with proper modifications, the approaches proposed in [48–50] can benefit from our results as well.

Solutions making these assumptions may result in unrealistic energy consumption measurements, especially for networks with higher data rates than environmental monitoring applications (ground temperature monitoring or ultraviolet monitoring). Since the sampling rate at which parameters such as temperature and/or ultraviolet radiation are sensed is low, the number of bits transmitted per second by individual nodes is small. Accordingly, there will be more room to efficiently reduce the idle listening time. This reduces the amount of energy wasted on idle listening, in which nodes wait for potentially incoming messages, while still maintaining a reasonable throughput.

7. Conclusions

In this paper, we have provided a mathematical framework to determine the optimal cluster size that maximizes the lifetime of sensor networks by minimizing the total energy expenditure in these networks. Three cluster-based protocols, namely, LEACH, LEACH-Coverage, and DBS comprise our target communication protocols, where all sensors communicate data through the cluster heads to the base station in a decentralized fashion. The analytical results are listed in a closed-form manner for several widely-used network configurations.

The analytical results show that:

- Under certain situations the optimal number of clusters can be independent of the network size.
- The energy consumption of the transmitter circuitry has no impact on the optimal cluster size, and
- The energy consumption of the receiver electronics can substantially change the optimal number of clusters and more importantly, it can make a decision on whether or not it is worth to perform clustering.

Extensive simulations on different network configurations are used to validate our analysis on the energy-efficient size for clusters. It has been shown that the outcomes of simulations are in complete accordance with analytical closed-form results. Further, cluster number variability is identified and accounted for as one of the problematic factors that can deteriorate the reliability and energy-efficiency of the studied decentralized protocols.

Appendix A

A number of cluster-based communication protocols take chain-based approaches to convey cluster heads' data to the base station. As an example, MR-LEACH [52] pursues multi-hop routing from cluster heads to a base station to conserve energy.

For such multi-hop schemes, we perform the analysis to find the optimal number of clusters. We assume that N sensor nodes are uniformly deployed within a circular field of radius R. The base station is located at the center of the sensing field. We also hypothetically partition the circular field into m separate concentric segments (also known as corona, annulus, or ring in the literature) considering $m + 1$ concentric circles whose center is the BS. The radius of concentric circles is represented by $r_0, r_1, r_2, \ldots, r_m = R$. In each round of the network operation, the network is divided into k_{opt} clusters and the cluster heads forward their aggregated data to the BS in a hop by hop fashion via cluster heads from closer segments. We denote segment i by S_i and we interpret S_0 as the BS. Hence, data traffic is forwarded hop by hop from S_i to S_{i-1} and so on until it reaches the BS.

Each non-cluster head node transmits its data to the cluster head once per round. Accordingly, the energy dissipation for non-cluster head nodes can be written as:

$$E_{non\text{-}CH} = lE_{elec}^{Tx} + l\varepsilon_{amp} d_{toCH}^{n}. \quad (64)$$

The expected value of the nth power of distance between cluster members and cluster heads is as follows:

$$E[d_{toCH}^{n}] = \frac{2}{n+2}\left(\frac{R}{\sqrt{k_{opt}}}\right)^{n}. \quad (65)$$

By substituting (64) into (65), the energy used in each non-cluster head node per round is given by:

$$E_{non\text{-}CH} = lE_{elec}^{Tx} + l\varepsilon_{amp}\frac{2}{n+2}\left(\frac{R}{\sqrt{k_{opt}}}\right)^{n}. \quad (66)$$

Each cluster head dissipates energy receiving signals from the nodes, aggregating the signals, forwarding the incoming data to the inner segment, and transmitting the aggregated data to the inner segment [53]. Hence, in each round, the energy dissipated in a cluster head node located in the ith segment becomes:

$$E_{CH}(S_i) = \left(\frac{N}{k_{opt}} - 1\right)lE_{elec}^{Rx} + \frac{N}{k_{opt}}lE_{DA} + l\frac{\sum_{j=i+1}^{m} S_j}{S_i}E_{elec}^{Rx} + l\frac{\sum_{j=i}^{m} S_j}{S_i}E_{elec}^{Tx} + l\frac{\sum_{j=i}^{m} S_j}{S_i}\varepsilon_{amp} d_{toS_{i-1}}^{n}, \quad (67)$$

where i resides in the interval $[1, m]$, i.e., $1 \leqslant i \leqslant m$. In case $i = 1$, the energy consumption of a cluster head located in S_1, which is the potential hot spot due to the fact that it carries the most relay traffic across the sensing field, can be calculated as:

$$E_{CH}(S_1) = \left(\frac{N}{k_{opt}} - 1\right)lE_{elec}^{Rx} + \frac{N}{k_{opt}}lE_{DA} + l\frac{R^2 - r_1^2}{r_1^2 - r_0^2}E_{elec}^{Rx} + l\frac{R^2 - r_0^2}{r_1^2 - r_0^2}E_{elec}^{Tx} + l\frac{R^2 - r_0^2}{r_1^2 - r_0^2}\varepsilon_{amp} r_1^{n}. \quad (68)$$

If we assume that the energy being dissipated to run the transmitter's circuitry is equal to the one consumed by the receiver's circuitry, i.e., $E_{elec}^{Tx} = E_{elec}^{Rx} = E_{elec}$, (68) can be further simplified:

$$E_{CH}(S_1) = \left(\frac{N}{k_{opt}} - 1\right)lE_{elec} + \frac{N}{k_{opt}}lE_{DA} + l\frac{2R^2 - r_1^2 - r_0^2}{r_1^2 - r_0^2}E_{elec} + l\frac{R^2 - r_0^2}{r_1^2 - r_0^2}\varepsilon_{amp} r_1^{n}. \quad (69)$$

It is obvious that the network lifetime (E_{total}/E_{round}) is dominated by the lifetime of S_1. For simplicity, we use $p_i = \frac{S_i}{\pi(R^2 - r_0^2)}$ $(1 \leqslant i \leqslant m)$ to denote the area percentage. Hence, in each data gathering round, the energy consumed in S_1 is given as follows:

$$E(S_1) = k_{opt}p_1\left(E_{CH}(S_1) + \left(\frac{N}{k_{opt}} - 1\right)E_{non\text{-}CH}\right). \quad (70)$$

Thus, the lifetime of area S_1 can be estimated as:

$$T = p_1 N E_0 / E(S_1), \quad (71)$$

where E_0 denotes the initial energy of each sensor node. The above equation is governed by two factors: the optimal number of clusters (k_{opt}) and the radii of C_1, i.e., r_1. Hence, in order to maximize T, one should minimize the function $f(r_1, k_{opt})$:

$$f(r_1, k_{opt}) = 2E_{elec}\frac{R^2}{r_1^2}k_{opt} - E_{elec}k_{opt} + \varepsilon_{amp}R^2 r_1^{n-2}k_{opt} + \frac{2}{n+2}\varepsilon_{amp}N\frac{R^n}{k_{opt}^{n/2}}. \quad (72)$$

Taking the partial derivatives yields:

$$\begin{cases} \frac{\partial f(r_1, k_{opt})}{\partial r_1} = -4E_{elec}R^2 r_1^{-3}k_{opt} + (n-4)\varepsilon_{amp}R^2 r_1^{2}k_{opt}, \\ \frac{\partial f(r_1, k_{opt})}{\partial k_{opt}} = E_{elec}\frac{2R^2 - r_1^2}{r_1^2} + \varepsilon_{amp}R^2 r_1^{n-2} - \frac{n}{n+2}\varepsilon_{amp}NR^n k_{opt}^{-\frac{n+2}{2}}. \end{cases} \quad (73)$$

It is worth noting that for a given E_{elec}, n, and ε_{amp}, we can determine r_1 and k_{opt} in such a way that $f(r_1, k_{opt})$ becomes minimized, in other words T is maximized. Examining the function $\partial f(r_1, k_{opt})/\ \partial f(r_1)$, we can find the value of r_1 for which the function becomes zero for $n > 2$. The fact that the function is decreasing for $n = 2$ shows that the optimal value of r_1 is:

$$r_{1\_opt} = \begin{cases} \sqrt[n]{\frac{4E_{elec}}{(n-2)\varepsilon_{amp}}} & 2 < n \leqslant 6, \\ \text{transmission radius} & n = 2. \end{cases} \quad (74)$$

Finally, we can obtain the optimal value of k_{opt} by finding r_{1_opt} and nulling the function $\partial f(r_1, k_{opt})/\partial f(k_{opt})$. With the optimal value of k_{opt} and properly choosing the size of r_1, we can achieve the optimized lifetime of S_1, hence, maximize the lifetime of the whole network.

References

[1] F. Zhao, J. Liu, J. Liu, L. Guibas, J. Reich, Collaborative signal and information processing: an information-directed approach, Proceedings of IEEE 91 (8) (2003) 1199–1209.
[2] D. Dardari, A. Conti, C. Buratti, R. Verdone, Mathematical evaluation of environmental monitoring estimation error through energy-efficient wireless sensor networks, IEEE Transactions on Mobile Computing 6 (7) (2007) 790–802.
[3] J.N. Al-Karaki, A.E. Kama, Routing techniques in wireless sensor networks: a survey, IEEE Journal of Wireless Communications 11 (6) (2004) 6–28.
[4] K. Ni, N. Ramanathan, M. Chehade, L. Balzano, S. Nair, S. Zahedi, G. Pottie, M. Hansen, M. Srivastava, Sensor network data fault types, ACM Transactions on Sensor Networks 5 (3) (2009) (Article 25).
[5] W.B. Heinzelman, A.P. Chandrakasan, H. Balakrishnan, Energy-efficient communication protocol for wireless microsensor networks, in: Proceedings of the 33th IEEE International Conference on System Sciences, pp. 1–10.
[6] W.B. Heinzelman, A.P. Chandrakasan, H. Balakrishnan, An application-specific protocol architecture for wireless microsensor networks, IEEE Transactions on Wireless Communications 1 (4) (2002) 660–670.
[7] Y.R. Tsai, Coverage-preserving routing protocols for randomly distributed wireless sensor networks, IEEE Transactions on Wireless Communications 6 (4) (2007) 1240–1245.
[8] N. Amini, A. Vahdatpour, F. Dabiri, H. Noshadi, M. Sarrafzadeh, Joint consideration of energy-efficiency and coverage-preservation in microsensor networks, Wireless Communications and Mobile Computing (2009). <http://dx.doi.org/10.1002/wcm.852>.
[9] C. Alippi, R. Camplani, M. Roveri, An adaptive LLC-based and hierarchical power-aware routing algorithm, IEEE Transactions on Instrumentation and Measurement 58 (9) (2009) 3347–3357.
[10] A. Khadivi, M. Shiva, FTPASC: a fault tolerant power aware protocol with static clustering for wireless sensor networks, in: Proceedings of the IEEE International Conference on Wireless and Mobile Computing, Networking and Communications, 2006, pp. 397–401.
[11] T.J. Chan, C.M. Chen, T.R. Chen, Optimal cluster number selection in ad-hoc wireless sensor networks, WSEAS Transactions on Communications 7 (8) (2008) 837–846.
[12] D. Kumar, C.A. Trilok, R.B. Patel, EEHC: energy efficient heterogeneous clustered scheme for wireless sensor networks, Computer Communication 32 (4) (2009) 662–667.
[13] L. Zhao, Q. Liang, An access-based low-energy hierarchy for sensor networks, in: Proceedings of the IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC), Barcelona, Spain, 2004.
[14] H. Kim, S.W. Kim, S. Lee, B. Son, Estimation of the optimal number of cluster-heads in sensor network, LNAI, 2005, pp. 87–94.
[15] Q.H. Wang, B.H. Zhao, Protocol for the application of cooperative MIMO based on clustering in sparse wireless sensor networks, Journal of China Universities of Posts and Telecommunications 14 (2) (2007) 51–57.
[16] T. Shu, M. Krunz, S. Vrudhula, Power balanced coverage-time optimization for clustered wireless sensor networks, in: Proceedings of the MobiHoc, 2005, pp. 111–120.
[17] N. Kim, S. Han, W.H. Kwon, Optimizing the number of clusters in multi-hop wireless sensor networks, IEICE Transactions on Communications E91-B (1) (2008) 318–321.
[18] I.C. Paschalidis, W. Lai, D. Starobinski, Asymptotically optimal transmission policies for large-scale low-power wireless sensor networks, IEEE/ACM Transactions on Networking 15 (1) (2007) 105–118.
[19] R. Madan, N.B. Mehta, A.F. Molisch, J. Zhang, Energy-efficient decentralized cooperative routing in wireless networks, IEEE Transactions on Automatic Control 54 (3) (2009) 512–527.
[20] S. Olariu, I. Stojmenovic, Design guidelines for maximizing lifetime and avoiding energy holes in sensor networks with uniform distribution and uniform reporting, in: Proceedings of IEEE INFOCOM 2006, 2006, pp. 1–12.
[21] L. Wang, A. Chen, S. Huang, A cross-layer investigation for the throughput performance of CSMA/CA-based WLANs with directional antennas and capture effect, IEEE Transactions on Vehicular Technology 56 (5) (2007) 2756–2766.
[22] H. Liu, L. Li, S. Jin, Cluster number variability problem in LEACH, in: Lecture Notes in Computer Science, 2006, pp. 429–437.
[23] Y. Wang, M. Xiong, Monte Carlo simulation of LEACH protocol for wireless sensor networks, in: Proceedings of the 6th International Conference on Parallel and Distributed Computing (PDCAT), 2005, pp. 85–88.
[24] B. Braem, B. Latre, I. Moerman, C. Blondia, E. Reusens, W. Joseph, L. Martens, P. Demeester, The need for cooperation and relaying in short-range high path loss sensor networks, in: Proceedings of 2007 International Conference on Sensor Technol. Appl. (SENSORCOMM 2007), pp. 566–571.
[25] Z. Cheng, M. Perillo, W.B. Heinzelman, General network lifetime and cost models for evaluating sensor network deployment strategies, IEEE Transactions on Mobile Computing 7 (4) (2008) 484–497.
[28] X. Wu, G. Chen, S. Das, Avoiding energy holes in wireless sensor networks with nonuniform node distribution, IEEE Transactions on Parallel and Distributed Systems 19 (5) (2008) 710–720.
[29] D. Herbert, G. Modelo-Howard, C. Perez-Toro, S. Bagchi, Fault tolerant ARIMA-based aggregation of data in sensor networks, in: Proceedings of the International Conference on Dependable Systems and Networks (DSN 2007).
[30] S. Soro, W.B. Heinzelman, Cluster head election techniques for coverage preservation in wireless sensor network, Ad Hoc Networks Journal 7 (5) (2009) 955–972.
[31] D. Wang, B. Xie, D.P. Agrawal, Coverage and lifetime optimization of wireless sensor networks with gaussian distribution, IEEE Transactions on Mobile Computing 7 (12) (2008) 1444–1458.
[32] L. Qing, Q.X. Zhu, M.W. Wang, Design of a distributed energy-efficient clustering algorithm for heterogeneous wireless sensor networks, Computer Communication 29 (12) (2006) 2230–2237.
[33] T. Banerjee, B. Xie, J.H. Jun, D.P. Agrawal, Increasing lifetime of wireless sensor networks using controllable mobile cluster heads, Wireless Communications and Mobile Computing (2009). <http://dx.doi.org/10.1002/wcm.763>.
[34] C. Hua, T.S.P. Yum, Optimal routing and data aggregation for maximizing lifetime of wireless sensor networks, IEEE/ACM Transactions on Networking 16 (4) (2008) 892–903.
[35] J. Chang, L. Tassiulas, Maximum lifetime routing in wireless sensor networks, IEEE/ACM Transactions on Networking 12 (4) (2004) 609–619.
[36] M. Bhardwaj, A. Chandrakasan, Coding under observation constraints, in: Proceedings of the Allerton Conference on Communication, Control, and Computing, Monticello, IL, September 2007, pp. 1082–1088.
[37] XBee ZigBee RF Modules datasheet. <http://www.digi.com>.
[38] Crossbow MICA2 datasheet: <http://www.xbow.com>.
[39] X. Wu, G. Chen, S. Das, Effects of denial-of-sleep attacks on wireless sensor network MAC Protocols, IEEE Transactions on Vehicular Technology 58 (1) (2009) 367–380.
[40] D. Yupho, J. Kabara, The effect of physical topology on wireless sensor network lifetime, Journal of Networks 2 (5) (2007) 14–23.
[42] ANT AT3 RF Transceiver Module datasheet. <http://www.thisisant.com>.
[43] TI EZ430-RF2500 Development Tool datasheet. <http://focus.ti.com>.
[44] W. Wu, M.A.T. Sanduleanu, X. Li, J.R. Long, 17 GHz front-ends for low-power wireless sensor networks, IEEE Journal Of Solid-State Circuits 43 (9) (2008) 1909–1919.
[45] J.W. Jung, A. Kailas, M.A. Ingram, E.M. Popovici, An evaluation of cooperation transmission considering practical energy models and passive reception, in: Proceedings of 1st International Symposium on Applied Sciences in Bio-Medical and Communication Technologies (ISABEL), October 25–28, 2008.
[46] M. Gribaudo, C.F. Chiasserini, R. Gaeta, M. Garetto, D. Manini, M. Sereno, A spatial fluid-based framework to analyze large-scale wireless sensor networks, in: Proceedings of the International Conference on Dependable Systems and Networks (DSN 2005), pp. 694–703.
[47] S. Olariu, I. Stojmenovic, Data-centric protocols for wireless sensor networks, in: I. Stojmenovic (Ed.), Handbook of Sensor Networks: Algorithms and Architectures, John Wiley & Sons, 2005, pp. 417–456.
[48] C. Li, M. Ye, G. Chen, J. Wu, An energy-efficient unequal clustering mechanism for wireless sensor networks, in: Proceedings of the 2nd IEEE International Conference on Mobile Ad-hoc and Sensor Systems (MASS), 2005.
[49] M.J. Handy, M. Haase, D. Timmermann, Low energy adaptive clustering hierarchy with deterministic cluster-head selection, in: Proceedings of the 4th IEEE International Conference on Mobile and Wireless Communications Networks, 2002, pp. 368–372.
[50] S. Soro, W.B. Heinzelman, Prolonging the lifetime of wireless sensor networks via unequal clustering, in: Proceedings of the 5th International Workshop on Algorithms for Wireless, Mobile, Ad Hoc and Sensor Networks (IEEE WMAN), 2005.
[51] M. Zuniga, B. Krishnamachari, An analysis of unreliability and asymmetry in low power wireless links, ACM Transactions on Sensor Networks 3 (2) (2007) 1–30.
[52] K. Srinivasan, P. Levis, RSSI is under appreciated, in: Proceedings of the Third Workshop on Embedded Networked Sensors (EmNets 2006), Cambridge, MA, May 2006.
[53] M.O. Farooq, A.B. Dogar, G.A. Shah, MR-LEACH: multi-hop routing with low energy adaptive clustering hierarchy, in: International Conference on Sensor Technologies and Applications, Venice, Italy, July 2010, pp. 262–268.