Solution, Midterm I – Version B

CSE 191
Oct. 3, 2014
12:00 - 12:50pm

First Name (Print):__________________________ Last Name (Print):__________________________

UB ID number:__________________________

1. This is a closed book, closed notes, and closed neighbor exam.
2. The logic equivalence rules and the logic inference laws (that you can use) are provided.
3. You must support your answer.
4. Print your name and UB ID number.
5. There are 6 problems and 25 points in this exam.
6. **Once the instructor announces "time's up", you must stop writing immediately. It's your responsibility to give your exam to TA within 2 mins.**

Name

1 (2+2 = 4 points). Let p, q, r be the following propositions:

p: Hiking is safe on the trail.

q: Berries are ripe along the trail.

r: Grizzly bears have been seen in the area.

Write the following propositions using p, q, r and logical operators.

1. Grizzly bears have been seen in the area and hiking on the trail is not safe, and berries are ripe along the trail.

$$r \wedge \neg p \wedge q$$

2. If berries are ripe along the trail, then hiking is safe only if grizzly bears have not been seen in this area.

$$q \to (p \to \neg r)$$

2 (4 points). Construct the truth table for the proposition $(\neg p \to t) \vee (s \wedge t)$.

p	t	s	$\neg p$	$\neg p \to t$	$s \wedge t$	$(\neg p \to t) \vee (s \wedge t)$
T	T	T	F	T	T	T
T	T	F	F	T	F	T
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	T	F	F	F
F	F	F	T	F	F	F

Table: Logic Equivalences

Equivalence	Name
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation laws
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \vee q) \equiv \neg p \wedge \neg q$ $\neg(p \wedge q) \equiv \neg p \vee \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	Negation laws

Logical Equivalences Involving Conditional Statements

$p \to q \equiv \neg p \vee q$

3 (4 points). **By using the rules of logical equivalences given above**, show the propositions $(s \to r) \wedge (q \to r)$ and $(s \vee q) \to r$ are logically equivalent. Namely show:

$$(s \to r) \wedge (q \to r) \equiv (s \vee q) \to r$$

LSH $\equiv (\neg s \vee r) \wedge (\neg q \vee r)$ — Substitution for "$\to$"
$\equiv (\neg s \wedge \neg q) \vee r$ — Distributive law
$\equiv \neg(s \vee q) \vee r$ — De Morgan's law
$\equiv (s \vee q) \to r$ — Substitution for "$\to$"
$\equiv$ RHS

4 (2+2 = 4 points). Rewrite the following proposition so that all negation symbols immediately precede predicates.

1. $\neg\exists y\forall x(P(x,y) \vee \neg Q(x))$

   $\forall y\exists x(\neg P(x,y) \wedge Q(x))$

2. $\neg\exists z(Q(z) \wedge \forall x\neg R(x,z))$

   $\forall z(\neg Q(z) \vee \exists x R(x,z))$

5 (4 points). Prove the following statement:

$\sqrt{3}$ is **irrational**.

Namely, you need to show $\sqrt{3} \neq \frac{a}{b}$ for any integers a, b.

Assume $\sqrt{3} = \frac{a}{b}$, without loss of generality, we can assume that a and b have no common divisors.

Then $a^2 = 3b^2$.

So a has divisor 3.

Then $a = 3k$ for some integer k.

So $3b^2 = (3k)^2 = 9k^2$

So $b^2 = 3k^2$

Then b must have divisor 3.

It contradicts the assumption that a and b have no common divisors.

So $\sqrt{3} \neq \frac{a}{b}$ for any inters a,b.

So $\sqrt{3}$ is irrational.

Table: Rules of Inference

Rule of Inference	Tautology	Name
p $\underline{p \to q}$ q	$(p \wedge (p \to q)) \to q$	**Modus ponens (MP)**
$\neg q$ $\underline{p \to q}$ $\neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	**Modus tonens (MT)**
$p \to q$ $\underline{q \to r}$ $p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	**Hypothetical syllogism (HS)**
$p \vee q$ $\underline{\neg p}$ q	$((p \vee q) \wedge \neg p) \to q$	**Disjunctive syllogism (DS)**
$\underline{p}$ $p \vee q$	$p \to (p \vee q)$	**Addition**
$\underline{p \wedge q}$ p	$(p \wedge q) \to p$	**Simplification**
p $\underline{q}$ $p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$	**Conjunction**
$p \vee q$ $\underline{\neg p \vee r}$ $q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$	**Resolution**

6 (5 points). Given premises: (1) $\neg q$, (2) $p \to q$, (3) $\neg p \to (s \vee q)$

Conclusion: s.

By using laws of inferences given above, write a valid logical argument that shows the premises lead to the conclusion.

(1) $\neg q$ premise

(2) $p \to q$ premise

(3) $\neg p$ Modus tonens from (1),(2)

(4) $\neg p \to (s \vee q)$ premise

(5) $s \vee q$ Modus ponens from (3),(4)

(6) s Disjunctive syllogism from(1),(5)