Midterm II – Version A

CSE 191

Solution

Nov 7, 2014

12:00 - 12:50pm

First Name (Print):________________________ Last Name (Print):________________________

UB ID number:________________________

1. This is a closed book, and closed neighbor exam. You may use a calculator, and a two-sided sheet of notes.
2. Support your answer.
3. Write your name on the top right-hand corner of every page.
4. There are 6 problems and 25 points in this exam.
5. **Once the instructor announces "time's up", you must stop writing immediately. It's your responsibility to give your exam to TA within 2 mins.**

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1 ($2 + 2 = 4$ points).

(a) Let A and B be two sets. Suppose that:

- $A - B = \{1, 5, 7, 8\}$;
- $B - A = \{2, 10\}$;
- $A \cap B = \{3, 6, 9\}$.

What is A and B?

$A = (A - B) \cup (A \cap B) = \{1, 3, 5, 6, 7, 8, 9\}$
$B = (B - A) \cup (A \cap B) = \{2, 3, 6, 9, 10\}$

(b) Let A, B, C be three sets. Suppose $A \cap C = B \cap C$. Can you conclude $A = B$? If your answer is "yes", prove it. If your answer is "no", give a counter example.

No. E.g. $A = \{1, 2\}, B = \{1, 3\}, C = \{1\}$

2 (3 points). Let A be the set of binary strings that satisfy the following conditions:

- The length of the binary string is 8.
- Either the first bit is 0; **or** the last two bits are 11.

Determine $|A|$. (Namely, determine the number of elements in A.)

Let A_1 be the set of 8-bit long binary strings whose first bit is 0. $|A_1| = 2^7$
Let A_2 be the set of 8-bit long binary strings whose last two bits are 11. $|A_2| = 2^6$
Then $A_1 \cap A_2$ is the set of 8-bit long binary strings whose first bit is 0 and last two bits are 11. $|A_1 \cap A_2| = 2^5$
Then $A_1 \cup A_2$ is the set of 8-bit long binary strings whose first bit is 0 or last two bits are 11.
$|A| = |A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2| = 2^7 + 2^6 - 2^5 = 160$

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Note: For problem 3, the solutions **may** involve factorial, combinations, permutations and powers, such as $6!$, $C(6,3)$, $P(10,5)$, or $(-5)^k$. In such cases, your solutions must be given in terms of these expressions. The numerical solution is not enough, and is not required. For example, if the solution is $P(7,3) \times 2^5$, you can just write it this way, or $7 \times 6 \times 5 \times 2^5$. Its numerical value 6720 is neither enough, nor required.

3 (6 points). How many ways can a photographer at a wedding arrange 6 people (including the bride and the groom) in a row, if:

(a) the bride is next to the groom?

Choose 2 neighboring positions for bride and groom, 5 ways.
Arrange bride and groom in these 2 positions, 2 ways (bride either on the left or the right side of the groom).
Arrange other 4 people, $P(4,4)$ ways.
So, there are $5 \times 2 \times P(4,4) = 240$ ways.

(b) the bride is NOT next to the groom?

The number of all the possible ways to arrange 6 people is $P(6,6)$.
Remove the ways that bride is next to the groom, which is $5 \times 2 \times P(4,4)$ ways.
So, there are $P(6,6) - 5 \times 2 \times P(4,4) = 480$ ways.

(c) the bride is positioned somewhere to the left of the groom?
(Hint: a row satisfies this requirement if and only if the reverse of the row does NOT satisfy this requirement).

Solution 1: There are $P(6,6)$ ways to arranged 6 people. Exactly half of these arrangements satisfy the requirement. So the answer is $P(6,6)/2 = 360$.

Solution 2: Choose 2 positions for bride and groom, $C(6,2)$ ways.
Arrange bride and groom in these 2 positions, 1 way (bride on the left side of the groom).
Arrange other 4 people, $P(4,4)$ ways.
So, there are $C(6,2) \times 1 \times P(4,4) = 360$ ways.

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4 ($3 + 3 = 6$ points). For this problem, you only need to give answer. **Support is NOT required**.

Let R be the set of real numbers; R^+ be the set of positive real numbers; Z be the set of integers; Z^+ be the set of positive integers.

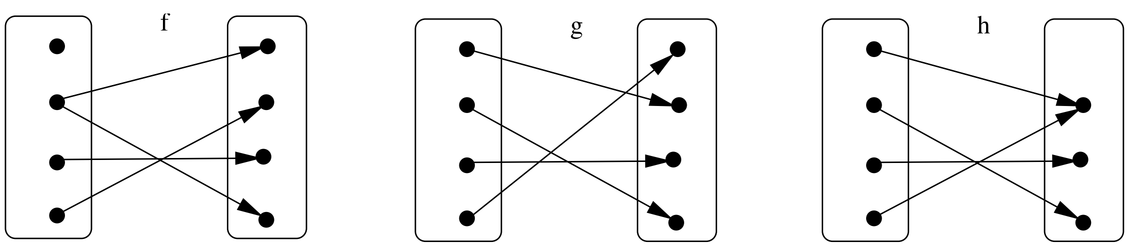
(a) Consider the following functions:

- $f : R \to R : \; f(x) = 2x - 1$.
- $g : Z \to Z : \; g(x) = 2x - 1$.
- $h : R^+ \to R : \; h(x) = x^2 - 1$.

Fill the following table by "yes" or "no":

	1-to-1?	onto?
f	yes	yes
g	yes	no
h	yes	no

(b) Let f, g and h be the functions represented by the following arrow diagrams.



Fill the following table by "yes" or "no":

	function?	1-to-1?	onto?
f	no	–	–
g	yes	yes	yes
h	yes	no	yes

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5 (2 points) Determine the cardinality of the following sets. (Namely: is it finite? countable infinite? uncountable?)

- $Z^+ \times Z^+ = \{(a, b) \mid a \text{ and } b \text{ are positive integers } \}$

  Countable infinite

- $(0, 0.02) = \{x \in R \mid 0 < x < 0.02\}$ = the set of real numbers between 0 and 0.02

  Uncountable

- $P(Z^+)$ = the power set of positive integers. (Namely the set of subsets of positive integers.)

  Uncountable

- The number of atoms in the solar system.

  Finite

6 (2+1+1 = 4 points).

(a) Find the value of the following sum:

$$\sum_{i=0}^{6}(3^i - 3 \cdot 2^i)$$

You must use the summation formula, (not by calculating the sum term by term.)

$\sum_{i=0}^{6}(3^i - 3 \cdot 2^i) = \sum_{i=0}^{6}(3^i) - 3\sum_{i=0}^{6}(2^i) = \frac{3^7-1}{3-1} - 3 \cdot \frac{2^7-1}{2-1} = 712$

(b) Find the value of the following sum: $\sum_{i=0}^{\infty}(2/3)^i = 1 + (2/3)^1 + (2/3)^2 + (2/3)^3 + \cdots$

$\sum_{i=0}^{\infty}(2/3)^i = \frac{(\frac{2}{3})^{\infty}-1}{\frac{2}{3}-1} = \frac{0-1}{\frac{2}{3}-1} = 3$

(c) Convert the periodic decimal x =0.32405405405$\cdots$ = to a fraction.

Solution 1 (using the method discussed in class: $=0.32\dot{4}0\dot{5}=\frac{32405-32}{99900}$.

Solution 2: $0.32\dot{4}0\dot{5} = 0.32 + 0.00405\sum_{i=0}^{\infty}(1/1000)^i = \frac{32}{100} + \frac{405}{100000} \cdot \frac{1}{1-1/1000} = \frac{32373}{99900} = \frac{1199}{3700}$