Midterm II – Version B

CSE 191

Solution

Nov 7, 2014

12:00 - 12:50pm

First Name (Print):________________________ Last Name (Print):________________________

UB ID number:________________________

1. This is a closed book, and closed neighbor exam. You may use a calculator, and a two-sided sheet of notes.
2. Support your answer.
3. Write your name on the top right-hand corner of every page.
4. There are 6 problems and 25 points in this exam.
5. **Once the instructor announces "time's up", you must stop writing immediately. It's your responsibility to give your exam to TA within 2 mins.**

Name

1 (2 + 2 = 4 points).

(a) Let A and B be two sets. Suppose that:

- $A - B = \{1, 5, 7, 8\}$;
- $B - A = \{3, 6, 9\}$;
- $A \cap B = \{2, 10\}$.

What is A and B?

$A = (A - B) \cup (A \cap B) = \{1, 2, 5, 7, 8, 10\}$;
$B = (B - A) \cup (A \cap B) = \{2, 3, 6, 9, 10\}$;

(b) Let A, B, C be three sets. Suppose $A \cup C = B \cup C$. Can you conclude $A = B$? If your answer is "yes", prove it. If your answer is "no", give a counter example.

No.
Counter example:
$A = \{1\}$, $B = \{1, 2\}$, $C = \{1, 2, 3\}$;
$A \cup C = B \cup C$, but $A \neq B$.

2 (3 points). Let A be the set of binary strings that satisfy the following conditions:

- The length of the binary string is 7.
- Either the first two bits are 00; **or** the last bit is 1.

Determine $|A|$. (Namely, determine the number of elements in A.)

Let B be the set of 7-bit binary strings of which the first two bits are 00;
Let C be the set of 7-bit binary strings of which the last bit is 1.
Then $|B| = 2^5 = 32, |C| = 2^6 = 64$.
Note that $B \cap C$ is the set of 7-bit strings whose first two bits are 00 and last bit is 1.
So $|B \cap C| = 2^4 = 16$.
So $|A| = |B \cup C| = |B| + |C| - |B \cap C| = 80$.

Name

Note: For problem 3, the solutions **may** involve factorial, combinations, permutations and powers, such as 6!, $C(6,3)$, $P(10,5)$, or $(-5)^k$. In such cases, your solutions must be given in terms of these expressions. The numerical solution is not enough, and is not required. For example, if the solution is $P(7,3) \times 2^5$, you can just write it this way, or $7 \times 6 \times 5 \times 2^5$. Its numerical value 6720 is neither enough, nor required.

3 (6 points). How many ways can a photographer at a wedding arrange 7 people (including the bride and the groom) in a row, if:

(a) the bride is NOT next to the groom?

The total number of possible arrangements: $P(7,7)$.
The total number of arrangements that the bride is at the left and next to the groom: $P(6,6)$.
The total number of arrangements that the bride is at the right and next to the groom: $P(6,6)$.
So the answer is: $P(7,7) - 2 \times P(6,6)$

(b) the bride is next to the groom?

The total number of arrangements that the bride is at the left and next to the groom: $P(6,6)$.
The total number of arrangements that the bride is at the right and next to the groom: $P(6,6)$.
So the answer is: $2 \times P(6,6)$

(c) the bride is positioned somewhere to the left of the groom?
(Hint: a row satisfies this requirement if and only if the reverse of the row does NOT satisfy this requirement).

Solution 1: There are $P(7,7)$ ways to arranged 7 people. Exactly half of these arrangements satisfy the requirement. So the answer is $P(7,7)/2$.

Solution 2: Choose 2 positions for bride and groom, $C(7,2)$ ways.
Arrange bride and groom in these 2 positions, 1 way (bride on the left side of the groom).
Arrange other 4 people,$P(5,5)$ ways.
So, there are $C(7,2) \times 1 \times P(5,5) = P(7,7) \cdot \frac{1}{2}$ ways.

Name

4 (3 + 3 = 6 points). For this problem, you only need to give answer. **Support is NOT required**.

Let R be the set of real numbers; R^+ be the set of positive real numbers; Z be the set of integers; Z^+ be the set of positive integers.

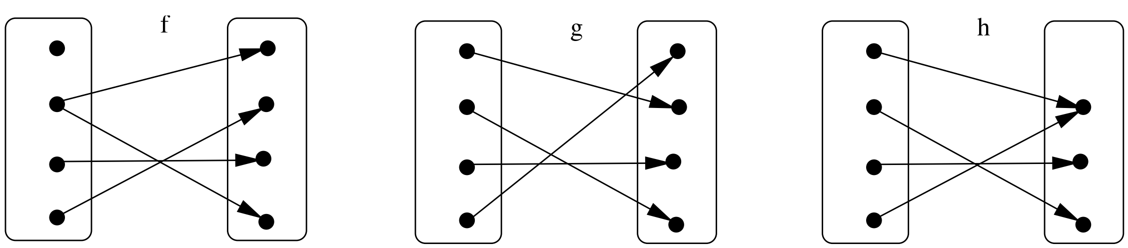
(a) Consider the following functions:

- $f : R \to R : \ f(x) = 3x - 1.$
- $g : Z \to Z : \ g(x) = 3x - 1.$
- $h : R^+ \to R : \ h(x) = x^2 + 1.$

Fill the following table by "yes" or "no":

	1-to-1?	onto?
f	yes	yes
g	yes	no
h	yes	no

(b) Let f, g and h be the functions represented by the following arrow diagrams.



Fill the following table by "yes" or "no":

	function?	1-to-1?	onto?
f	no	-	-
g	yes	yes	yes
h	yes	no	yes

Note: because f is not a function, you don't need to answer the 2nd and 3rd columns of the 1st row.

Name

5 (2 points) Determine the cardinality of the following sets. (Namely: is it finite? countable infinite? uncountable?)

- $(0, 0.02) = \{x \in R \mid 0 < x < 0.02\}$ = the set of real numbers between 0 and 0.02

  uncountable infinite

- $P(Z^+)$ = the power set of positive integers. (Namely the set of subsets of positive integers.)

  uncountable infinite

- $Z^+ \times Z^+ = \{(a, b) \mid a$ and b are positive integers $\}$

  countable infinite

- The number of atoms in the solar system.

  finite

6 (2+1+1 = 4 points).

(a) Find the value of the following sum:

$$\sum_{i=0}^{6}(4^i - 4 \cdot 2^i)$$

You must use the summation formula, (not by calculating the sum term by term.)

$$\sum_{i=0}^{6}(4^i - 4 \cdot 2^i) = \sum_{i=0}^{6}(4^i) - \sum_{i=0}^{6}(4 \cdot 2^i) = \frac{4^7 - 1}{4 - 1} - 4 \cdot \frac{2^7 - 1}{2 - 1} = 4953$$

(b) Find the value of the following sum: $\sum_{i=0}^{\infty}(3/4)^i = 1 + (3/4)^1 + (3/4)^2 + (3/4)^3 + \cdots$

$$\sum_{i=0}^{\infty}(3/4)^i = \lim_{n\to\infty}\frac{(3/4)^n - 1}{3/4 - 1} = 4$$

(c) Convert the periodic decimal x =0.91328328328$\cdots = 0.91\dot{3}2\dot{8}$ to a fraction.

Solution 1 (using the formula discussed in class):
$0.91\dot{3}2\dot{8}$=$\frac{91328-91}{99900}$

Solution 2:

$$0.91\dot{3}2\dot{8} = 0.91 + 0.00328(1 + (1/1000)^2 + (1/1000)^3 + \cdots)$$

$$= \frac{91}{100} + 0.00328 \cdot \lim_{n\to\infty}\frac{(1/1000)^n - 1}{1/1000 - 1} = \frac{91}{100} + \frac{328}{99900} = \frac{91237}{99900}$$