Solution, Assignment #10, CSE 191

Fall, 2014
Solution

General Guidelines:
This assignment will NOT be collected, nor graded. **There will be NO quiz based on this assignment. However, there will be similar problems in Final exam.** So you should carefully complete this assignment as if it were to be graded.

1. (0 points). Page 581, Problem 4, (b), (c), (d).

(b): reflexive, symmetric, not antisymmetric, (a and b born in the same day. b and a also born in the same day. But a might not be b), transitive.

(c): reflexive, symmetric, not antisymmetric, (a and b have the same first name. b and a also have the same first name. But a might not be b), transitive.

(d): reflexive, symmetric, not antisymmetric,(a and b have the same grandmother. b and a also have the same grandmother. But a might not be b), not transitive.

2. (0 point). Page 581, Problem 6, (a), (c), (e).

(a):
- R is not reflexive: for any $x \neq 0$, $(x, x) \notin R$ (because $x + x \neq 0$).
- R is symmetric: $(x, y) \in R \Leftrightarrow x + y = 0 \Leftrightarrow y + x = 0 \Leftrightarrow (y, x) \in R$.
- R is not antisymmetric: $(3, -3) \in R$ and $(-3, 3) \in R$ but $3 \neq -3$.
- R is not transitive: $(3, -3) \in R$ and $(-3, 3) \in R$. But $(3, 3) \notin R$ because $3 + 3 \neq 0$.

(c):
- R is reflexive: $\forall x$, $(x, x) \in R$ because $x - x = 0$ is a rational number.
- R is symmetric: $(x, y) \in R \Leftrightarrow x - y = r$ is a rational number; which implies $y - x = -r$ is a rational number; which implies $(y, x) \in R$.
- R is not antisymmetric: $(2 + \sqrt{3}, 3 + \sqrt{3}) \in R$ and $(3 + \sqrt{3}, 2 + \sqrt{3}) \in R$ but $2 + \sqrt{3} \neq 3 + \sqrt{3}$.
- R is transitive: $(x, y) \in R$ and $(y, z) \in R$ imply $x - y = r_1$ and $y - z = r_2$ are two rational numbers. Then $x - z = (x - y) + (y - z) = r_1 + r_2$ is rational (note that the sum of two rational numbers is rational). So $(x, z) \in R$.

(e):
- R is reflexive: $\forall x$, $x \cdot x \geq 0$ implies $(x, x) \in R$.
- R is symmetric: $(x, y) \in R \Leftrightarrow x \cdot y \geq 0 \Leftrightarrow y \cdot x \geq 0$ implies $(y, x) \in R$.
- R is not antisymmetric: $(3, 1) \in R$ and $(1, 3) \in R$ but $1 \neq 3$.
- R is not transitive: $(3, 0) \in R$ and $(0, -3) \in R$. But $(3, -3) \notin R$.

3. (0 point). Page 581, Problem 3, (a), (d).

(a):
- R is not reflexive since $(4, 4) \notin R$.
- R is not symmetric: $(2, 4) \in R$ but $(4, 2) \notin R$.
- R is not antisymmetric: $(2, 3) \in R$ and $(3, 2) \in R$ but $2 \neq 3$.
- R is transitive.

(c):
- R is not reflexive: $(2, 2) \notin R$.
- R is symmetric.
- R is not antisymmetric: $(2, 4) \in R$ and $(4, 2) \in R$, but $2 \neq 4$.

- R is not transitive: $(2,4) \in R$ $(4,2) \in R$. But $(2,2) \notin R$.

(d):
- R is not reflexive: $(1,1) \notin R$.
- R is not symmetric: $(1,2) \in R$, but $(2,1) \notin R$.
- R is antisymmetric.
- R is not transitive: $(1,2) \in R$ and $(2,3) \in R$, but $(1,3) \notin R$.

4. (0 point). Page 582, Problem 32.
$S \circ R = \{(1,1),(1,2),(2,1),(2,2)\}$

5. (0 point). Page 582, Problem 34 (a), (c)

(a): $R_1 \cup R_3 = \{(a,b) \in R^2 \mid a > b \text{ or } a < b\} = \{(a,b) \in R^2 \mid a \neq b\} = R^2 - \{(a,a) \mid a \in R\} = R_6$.

(c): $R_2 \cap R_4 = \{(a,b) \in R^2 \mid a \geq b \text{ and } a \leq b\} = \{(a,b) \in R^2 \mid a = b\} = R_5$.

6. (0 point). Page 582, Problem 36 (c), (e)

(c): $(a,c) \in R_1 \circ R_3 \Leftrightarrow \exists b$ such that $(a,b) \in R_3$ and $(b,c) \in R_1 \Leftrightarrow \exists b$ such that $a < b$ and $b > c$. Since the last condition is always true, we have $(a,b) \in R_1 \circ R_3$ for any a, b. Thus $R_1 \circ R_3 = R^2$.

(e): $(a,c) \in R_1 \circ R_5 \Leftrightarrow \exists b$ such that $(a,b) \in R_5$ and $(b,c) \in R_1 \Leftrightarrow \exists b$ such that $a = b$ and $b > c$. $\Leftrightarrow a > c$.
Thus $R_1 \circ R_5 = \{(a,c) |\ a > c\} = R_1$.

7. (0 point). Page 596, Problem 14 (a), (b), (c), (d).

(a): $M_{R_1} \cup M_{R_2} = M_{R_1} \vee M_{R_2} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

(b): $M_{R_1} \cap M_{R_2} = M_{R_1} \wedge M_{R_2} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

(c): $M_{R_2 \circ R_1} = M_{R_1} \odot M_{R_2} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

(d): $M_{R_1 \circ R_1} = M_{R_1} \odot M_{R_1} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

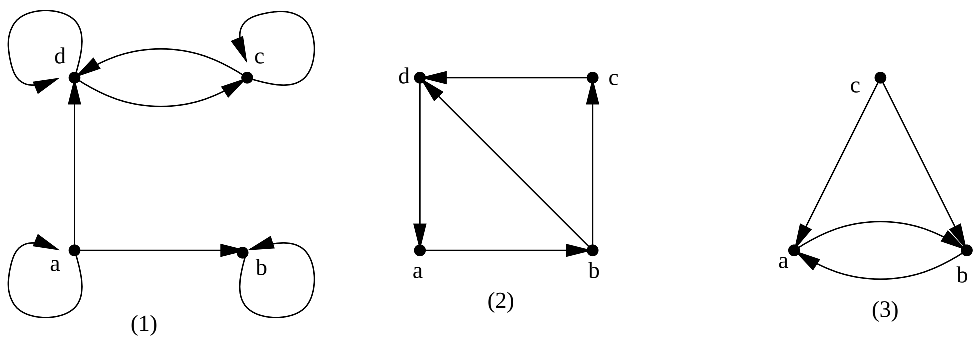
8. (0 point). For each of the relations represented by the following directed graph, determine if the relation is:
(i) reflexive? (ii) symmetric? (iii) antisymmetric? (iv) transitive?

(i):
- It's reflexive.
- It's not symmetric since $(a,d) \in R$ but $(d,a) \notin R$.
- It's not antisymmetric since $(c,d) \in R$ and $(d,c) \in R$ but $c \neq d$.
- It's not transitive since $(a,d),(d,c) \in R$ but $(a,c) \notin R$.

(ii):
- It's not reflexive since $(a,a),(b,b),(c,c)(d,d) \notin R$.

- It's not symmetric since $(a, b) \in R$ but $(b, a) \notin R$.
- It's antisymmetric.
- It's not transitive since $(a, b), (b, c) \in R$ but $(a, c) \notin R$.

(iii):
- It's not reflexive since $(a, a) \notin R$.
- It's not symmetric since $(c, a) \in R$ but $(a, c) \notin R$.
- It's not antisymmetric since $(a, b) \in R$ and $(b, a) \in R$ but $a \neq b$.
- It's not transitive: $(a, b) \in R$ and $(b, a) \in R$, but $(a, a) \notin R$.



9. (0 point). Page 615, Problem 2 (a), (b), (d).
(You ONLY need to decide whether each of the relations is an equivalence relation or not. You don't need to do anything else.)

(a): yes.

(b): yes.

(d): no, not transitive. a and b have met. b and c have met. It doesn't imply a and c have meet.

10. (0 point). Page 615, Problem 16. What are the equivalence classes of this relation?

reflexive: for any integer pair (a, b), $ab = ba$. So, $((a, b), (a, b)) \in R$.

symmetric: for any two ordered pairs $((a, b), (c, d)) \in R$, we have $ad = bc \Leftrightarrow cb = da$. It implies $((c, d), (a, b)) \in R$.

transitive: for any $((a, b), (c, d)) \in R$ and $((c, d), (e, f)) \in R$, we have $ad = bc \Leftrightarrow a/b = c/d$ and $cf = de \Leftrightarrow c/d = e/f$. This implies $a/b = e/f \Leftrightarrow af = be$. This implies $((a, b), (e, f)) \in R$.

$[(a, b)]_R = \{(c, d) \mid ad = bc\} = \{(c, d) \mid a/bd = c/d\}$, which corresponds to the set of rational numbers whose value is a/b.
For example: $(2, 3)]_R = \{(2, 3), (4, 6), (6, 9), (8, 12) \ldots\}$

11. (0 point). Page 630, Problem 2 (b), (c), (d).

(b): It's a partial order.

(c): It's not a partial order. Not transitive. $(3, 1), (1, 2) \in R$ but $(3, 2) \notin R$.

(d): It's not a partial order. Not transitive since $(1, 3), (3, 0) \in R$ but $(1, 0) \notin R$.