Solution of Assignment #2, CS/191
Fall, 2014

1. Page 35, problem 10,
(0 points) (b)

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Since $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is always T, it is a tautology.

(0 points) (c)
sol:

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since $[p \wedge (p \to q)] \to q$ is always T, it is a tautology.

(0 points) (d)

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge (p \to r) \wedge (q \to r)$	$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

Since $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$ is always T, it is a tautology.

2. (0 points), Page 35, problem 14.
sol:

p	q	$p \to q$	$\neg p \wedge (p \to q)$	$[\neg p \wedge (p \to q)] \to \neg q$
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

So $[\neg p \wedge (p \to q)] \to \neg q$ is not a tautology.

3. (0 points), page 35, problem 18.

$$\begin{aligned}
& p \to q \\
\equiv & \neg p \lor q && \text{by the implication law (the first law in Table 7.)} \\
\equiv & q \lor (\neg p) && \text{by commutative laws} \\
\equiv & \neg(\neg q) \lor (\neg p) && \text{by double negation law} \\
\equiv & \neg q \to \neg p && \text{by implication law}
\end{aligned}$$

4. (0 points), page 35, problem 23.

$$\begin{aligned}
& (p \lor q) \to r \\
\equiv & \neg(p \lor q) \lor r && \text{by implication law} \\
\equiv & (\neg p \land \neg q) \lor r && \text{by de Morgan's laws} \\
\equiv & (\neg p \lor r) \land (\neg q \lor r) && \text{by distributive laws} \\
\equiv & (p \to r) \land (q \to r) && \text{by implication laws twice}
\end{aligned}$$

5. (0 points), page 35, problem 24.

$$\begin{aligned}
& (p \to q) \lor (p \to r) \\
\equiv & (\neg p \lor q) \lor (\neg p \lor r) && \text{by implication law, twice} \\
\equiv & (\neg p \lor \neg p) \lor (q \lor r) && \text{by commutative and associative laws} \\
\equiv & \neg p \lor (q \lor r) && \text{by Idempotent laws} \\
\equiv & p \to (q \lor r) && \text{by implication law}
\end{aligned}$$

6. (0 points), page 35, problem 26.

$$\begin{aligned}
& \neg p \to (q \to r) \\
\equiv & p \lor (q \to r) && \text{by implication law} \\
\equiv & p \lor (\neg q \lor r) && \text{by implication law} \\
\equiv & \neg q \lor (p \lor r) && \text{by commutative and associative laws} \\
\equiv & q \to (p \lor r) && \text{by implication law}
\end{aligned}$$

7. (0 points), page 35, problem 30. Show that $(p \lor q) \land (\neg p \lor r) \to (q \lor r)$ is a tautology.
sol:

$$\begin{aligned}
& (p \lor q) \land (\neg p \lor r) \to (q \lor r) \\
\equiv & \neg[(p \lor q) \land (\neg p \lor r)] \lor (q \lor r) && \text{by implication law} \\
\equiv & [\neg(p \lor q) \lor \neg(\neg p \lor r)] \lor (q \lor r) && \text{by de Morgan's law} \\
\equiv & [(\neg p \land \neg q) \lor (p \land \neg r)] \lor (q \lor r) && \text{by de Morgan's law} \\
\equiv & [(\neg p \land \neg q) \lor q] \lor [(p \land \neg r) \lor r] && \text{by commutative and associative laws} \\
\equiv & [(\neg p \lor q) \land (\neg q \lor q)] \lor [(p \lor r) \land (\neg r \lor r)] && \text{by distributive laws} \\
\equiv & (\neg p \lor q) \lor (p \lor r) && \text{by negation and identity laws} \\
\equiv & (\neg p \lor p) \lor (q \lor r) && \text{by communicative and associative laws} \\
\equiv & T && \text{by negation and domination laws}
\end{aligned}$$

8. (0 points), page 64, problem 6.

(d) sol: There is a student in your school who is enrolled in Math 222 and in CS 252.

(e) sol: There are two different students x and y such that if the student x takes the class z, then the student y also takes the class z for every classes z.

(f) sol: There are two different students x and y such that x takes z if and only if y takes z for all classes z.

9. page 65, problem 10,

(0 points)
(a) sol: $\forall x F(x, Fred)$
(b) sol: $\forall y F(Evelyn, y)$
(c) sol: $\forall x \exists y F(x, y)$
(e) sol: $\forall x \exists y F(y, x)$

(0 points)
(f) sol: $\neg \exists x (F(x, \text{Fred}) \wedge F(x, \text{Jerry}))$
(i) sol: $\neg \exists x F(x, x)$

10. page 67, problem 28,
(0 points)
(c) sol: True. Let x be 0.
(e) sol: True. Let $y = 1/x$
(f) sol: False

(0 points)
(g) sol: True. Let y be $1 - x$.
(i) sol: False. Let $x = 0$. Then $x + y = 2$ and $2x - y = 1$ imply $y = 2$ and $y = -1$. So for $x = 0$ there exists no y such that $x + y = 2$ and $2x - y = 1$.
(j) sol: True. Let $z = (x + y)/2$

11. page 67, problem 30,
(0 points)
(b) sol: $\exists x \forall y \neg P(x, y)$
(c) sol: $\forall y (\neg Q(y) \vee \exists x R(x, y))$

(0 points)
(d) sol: $\forall y (\forall x \neg R(x, y) \wedge \exists x \neg S(x, y))$
(e) sol: $\forall y (\exists x \forall z \neg T(x, y, z) \wedge \forall x \exists z \neg U(x, y, z))$