**Solution #5, CSE 191**
Fall, 2014

1. (0 points). Page 136, Prob 4. Let $A = \{a, b, c, d\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find
a) $A \cup B = \{a, b, c, d\} \cup \{a, b, c, d, e, f, g, h\} = \{a, b, c, d, e, f, g, h\}$.
b) $A \cap B = \{a, b, c, d\} \cap \{a, b, c, d, e, f, g, h\} = \{a, b, c, d\}$.
c) $A - B = \{a, b, c, d\} - \{a, b, c, d, e, f, g, h\} = \emptyset$.
d) $B - A = \{a, b, c, d, e, f, g, h\} - \{a, b, c, d\} = \{e, f, g, h\}$.

2. (0 points). Page 136, Prob 14.
sol:
$A = (A - B) \cup (A \cap B) = \{1, 5, 7, 8\} \cup \{3, 6, 9\} = \{1, 3, 5, 6, 7, 8, 9\}$.
$B = (B - A) \cup (A \cap B) = \{2, 10\} \cup \{3, 6, 9\} = \{2, 3, 6, 9, 10\}$.

3. (0 points). Page 136, Prob 18 (d) and (e)
sol:
d) $(A - C) \cap (C - B) = \emptyset$
Suppose $x \in$ LHS
$\Leftrightarrow x \in (A - C)$ and $x \in (C - B)$ by definition of intersection
$\Leftrightarrow (x \in A \wedge x \notin C)$ and $(x \in C \wedge x \notin B)$ by definition of set difference
$\Leftrightarrow (x \notin C$ and $x \in C)$
$\Leftrightarrow x \in C \cap \overline{C} = \emptyset$ by definition of intersection and complement law
$\Leftrightarrow$LHS=RHS.
e) $(B - A) \cup (C - A) = (B \cup C) - A$
$x \in$ LHS
$\Leftrightarrow x \in (B - A) \vee x \in (C - A)$ by definition of union
$\Leftrightarrow (x \in B \wedge x \notin A) \vee (x \in C \wedge x \notin A)$ by definition of set difference
$\Leftrightarrow (x \notin A) \wedge (x \in B \vee x \in C)$ by distributive law
$\Leftrightarrow (x \in B \vee x \in C) \wedge (x \notin A)$ by commutative law
$\Leftrightarrow (B \cup C) - A$ by definition of union and set difference
$\Leftrightarrow x \in$ RHS

4. (0 points). Page 136, Prob 26.
Sol: See Fig 1
Note: For (2), we have $\overline{A} \cap \overline{B} \cap \overline{C} = \overline{A \cup B \cup C}$ by De Morgan's law.

5. (0 points). Page 136, Prob 29.
a) B is a subset of A
b) A is a subset of B
c) B is a subset of $\overline{A}$; A and B are disjoint.
d) We can say nothing about B and A, as this property always holds true.
e) A=B

6. (0 points). Page 137, Prob 30.
Sol:

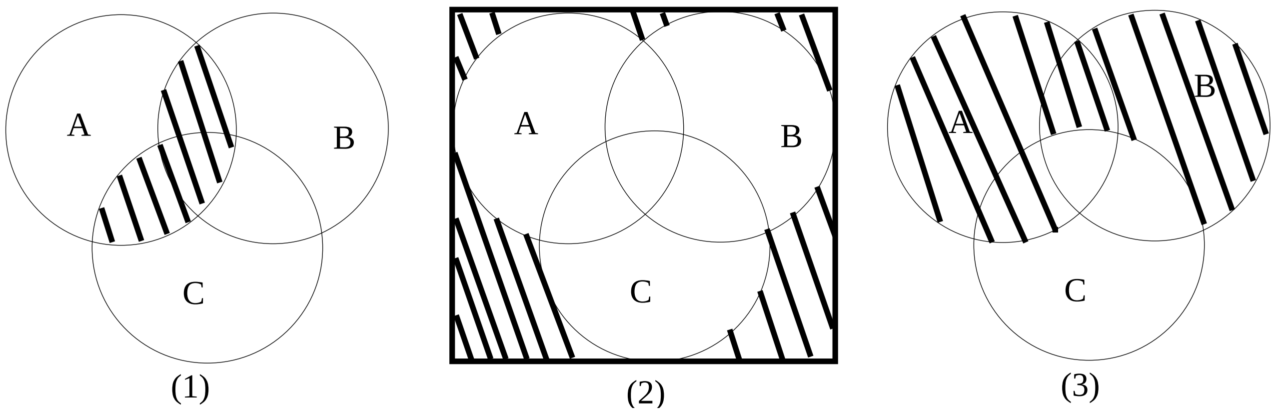


Figure 1: (1) $A \cap (B \cup C)$; (2) $\overline{A} \cap \overline{B} \cap \overline{C}$; (3)$(A - B) \cup (A - C) \cup (B - C)$.

(a) No. Counter example: $A = \{1, 2, 3\}, B = \{1, 2\}, C = \{3\}$. $A \cup C = B \cup C$. $A \neq B$.
(b) No. Counter example: $A = \{1, 2, 3\}, B = \{1, 2\}, C = \{2\}$. $A \cap C = B \cap C$. $A \neq B$.
(c) Yes. We show $A = B$ by showing $A \subseteq B$ and $B \subseteq A$.
To show $A \subseteq B$, consider any element $x \in A$. We need to show $x \in B$.

Case 1: $x \in C$. Then $x \in A \cap C = B \cap C$ (because the assumption $A \cap C = B \cap C$). Thus $x \in B$ as to be shown.

Case 2: $x \notin C$. We have $x \in A \cup C = B \cup C$ (because the assumption $A \cup C = B \cup C$). However, because the assumption that $x \notin C$ for this case, we must have $x \in B$ as to be shown.

Symmetrically, we can show $B \subseteq A$.

7. (0 points). Page 137, Prob 39.
What can you say about the sets A and B if $A \oplus B = A$?
Sol: The set B is the empty set $\emptyset$. If there were any elements in B that were also in A, then these elements would not be in the symmetric difference and the symmetric difference could not contain all of A. If there were any elements in B that were not in A, these elements would be included in the symmetric difference and therefore the symmetric difference could not be only A alone. Therefore, there must be no items in B.

8. (0 points). Page 137, Prob 52 (a) and (b)
Sol:
a) 0 0 1 1 1 0 0 0 0 0
b) 1 0 1 0 0 1 0 0 0 1

9. (0 points). Page 137, Prob 54.
sol:
a) $\emptyset$.
b) the entire set $U$.

10. (0 points) Page 137, Prob 56.
sol: Let $A$ and $B$ be two sets. Let $s_A$ and $S_B$ be the bit-string representation of $A$ and $B$ respectively. Then the symmetric difference $A \oplus B$ is represented by:
$s_A \oplus s_B$. (Here $\oplus$ is the bit-wise exclusive or operator. In C++, this operator is ˆ).

11. (0 points) sol: Let $A$ be the set of the students who have taken CSE 115, $B$ be the set of

students who have taken CSE 191 and $C$ be the set of students who have taken CSE 250.

Let $D$ be the students has taken at least one of CSE 115, CSE 191 and CSE 250.

$|D| = |A \cup B \cup C| = |A| + |B| + |C| - |(A \cap B)| - |(B \cap C)| - |(A \cap C)| + |(A \cap B \cap C)| = 96 + 86 + 65 - 46 - 35 - 35 + 15 = 146$.

So, the number of students not taken any of 115, 191 and 250 is the total number of students minus the number of students has taken at least one of CSE 115, 191 and 250; $150 - 146 = 4$.

12. (0 points). Calculate the number of binary strings of length 7, with the first two bits = 01, **OR** the last bit = 1.

Let $A$ = set of binary strings of length 7, with the first two bits = 01.

Let $B$ = set of binary strings of length 7, with the last bit = 1.

We need to calculate: $|A \cup B|$.

We have: $|A| = 2^{7-2} = 2^5 = 32$; $|B| = 2^{7-1} = 2^6 = 64$; and $|A \cap B| = 2^{7-3} = 2^4 = 16$.

Thus: $|A \cup B| = |A| + |B| - |A \cap B| = 32 + 64 - 16 = 80$.