Solution #7, CSE 191

Fall, 2014

1. (0 points). Page 153, Prob 10 and 11.

   sol:

   (a) f is both 1-to-1 and onto.

   (b) f is neither 1-to-1, nor onto.

   (c) f is neither 1-to-1, nor onto.

2. (0 points). Page 153, Prob 12 and 13.

   sol:

   (a) f is both 1-to-1 and onto.

   (b) f is neither 1-to-1, nor onto.

   (c) f is 1-to-1, but not onto.

   (d) f is not 1-to-1 but is onto.

3. (0 points). Page 153, Prob 20 (a), (b), (c), (d).

   sol:

   (a) $f(x) = x + 1$.

   (b) $f(x) = \lfloor \frac{x}{2} \rfloor$.

   (c) $f(x) = \begin{cases} x+1 & x \text{ is even,} \\ x-1 & x \text{ is odd.} \end{cases}$

   (d) $f(x) = \lfloor \frac{x}{2} \rfloor + 1$.

4. (0 points). Page 153, Prob 22 (b), (c), (d)

   sol:

   (b) no. $f(x) = -3x^2 + 7$ is not an one-to-one function since $f(1) = 4 = f(-1)$.

   (c) Strictly speaking, the function f is **not even a function from** R **to** R, because f is undefined at -2.

   If we consider the domain $R' = R - \{-2\}$. Then $f : R' \to R$ is:

   one to one: for any $x_1, x_2 \in \mathbf{R'}$ such that $f(x_1) = f(x_2)$, we need to show $x_1 = x_2$. We have: $f(x_1) = f(x_2) \Leftrightarrow \frac{x_1+1}{x_1+2} = \frac{x_2+1}{x_2+2} \Leftrightarrow x_1x_2 + 2x_1 + x_2 + 2 = x_1x_2 + x_1 + 2x_2 + 2 \Leftrightarrow x_1 = x_2$.

   but not onto: because $f(x) = \frac{x+1}{x+2} \neq 1$ for any $x \in R'$. So 1 has no pre-image for f.

   (d) yes.

   one to one: for any $x_1, x_2 \in \mathbf{R}$ such that $(x_1)^5 + 1 = (x_2)^5 + 1$, then we have $(x_1)^5 = (x_2)^5 \Leftrightarrow x_1 = x_2$.

onto: any $a \in \mathbf{R}$, $x^5 + 1 = a \Leftrightarrow x = \sqrt[5]{a-1}$.

5. (0 points). Page 154, Prob 30 (a), (b), (d).

   (a) $f(S) = \{1\}$.
   (b) $f(S) = \{-1, 1, 5, 9, 15\}$.
   (c) $f(S) = \{0, 1, 2\}$.
   (d) $f(S) = \{0, 1, 5, 16\}$.

6. (0 points). Page 154, Prob 32.

   sol:

   (a) even integers.
   (b) 0 or even positive integers.
   (c) $\mathbf{R}$.

7. (0 points). Page 154, Prob 36 and 37.

   sol:

   $f \circ g = x^2 + 4x + 5$

   $g \circ f = x^2 + 3$

   $(f + g)(x) = x^2 + x + 3$

   $(fg)(x) = x^3 + 2x^2 + x + 2$

8. (4 points). Page 167, Prob 6 (b), (c), (d), (e).

   sol:

   (b) 1, 3, 6, 10, 15, 21.
   (c) 1, 5, 19, 65, 211, 665.
   (d) $1, 1, 1, 2, 2, 2$.
   (e) $1, 5, 6, 11, 17, 28$.

9. (0 points). Page 168, Prob 10 (c), (d), (e).

   sol:

   (c) $a_0 = 1$, $a_1 = 3 \cdot 1^2 = 3$; $a_2 = 3 \cdot 3^2 = 27$;

   $a_3 = 3 \cdot (3^3)^2 = 3^7$;

   $a_4 = 3 \cdot (3^7)^2 = 3^{15}$;

   $a_5 = 3 \cdot (3^{15})^2 = 3^{31}$.

   (d) $a_0 = -1$;, $a_1 = 0$;

   $a_2 = 2 \cdot 0 + 1^2 = 1$;

   $a_3 = 3 \cdot 1 + 0^2 = 3$;

   $a_4 = 4 \cdot 3 + 1^2 = 13$;

   $a_5 = 5 \cdot 13 + 3^2 = 74$.

(e) $a_0 = 1$; $a_1 = 1$; $a_2 = 2$;

$a_3 = 2 - 1 + 1 = 2$;

$a_4 = 2 - 2 + 1 = 1$;

$a_5 = 1 - 2 + 2 = 1$.

10. (0 points). Page 168, Prob 16 (b), (c), (d), (e).

sol:

(b) $a_0 = 1$; $a_1 = 1 + 3 = 4$; $a_2 = 4 + 3 = 7$; $a_3 = 7 + 3 = 10 \ldots$

So $a_n = 3n + 1$.

(c) $a_0 = 4$; $a_1 = 4 - 1 = 3$; $a_2 = 3 - 2 = 1$, $a_3 = 1 - 3 = -2$; $a_4 = -2 - 4 = -6$; $a_5 = -6 - 5 = -11; \ldots$

So $a_n = 4 - \sum_{i=1}^{n} i = 4 - n(n+1)/2$.

(d) $a_0 = -1$; $a_1 = 2(-1) - 3 = -5$; $a_2 = 2(-5) - 3 = -13$; $a_3 = 2(-13) - 3 = -29$; $a_4 = 2(-29) - 3 = -61; \ldots.$

So $a_n = -2^{n+2} + 3$.

(e) $a_0 = 2$; $a_1 = 2 \cdot 2 = 4$; $a_2 = 3 \cdot 4 = 12$; $a_3 = 4 \cdot 12 = 48; \ldots.$

So $a_n = 2 \cdot (n+1)!$

11. (0 points). Page 168, Prob 18 (b), (c).

sol:

(b) $1000 * (1.09)^n$.

(c) $1000 * (1.09)^{100} = 5529040.791825879$

12. (0 points). Page 169, Prob 32 (c), (d).

sol:

(c)

$$\sum_{j=0}^{8}(2 \cdot 3^j + 3 \cdot 2^j) = 2 \cdot \frac{3^9 - 1}{3 - 1} + 3 \cdot \frac{2^9 - 1}{2 - 1} = 21216.$$

(d)

$$\sum_{j=0}^{8}(2^{j+1} - 2^j) = \sum_{j=0}^{8}\big(2^j \cdot (2-1)\big) = \frac{2^9 - 1}{2 - 1} = 511.$$

13. (0 points).

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

sol: As discussed in class, $\sum_{i=1}^{n} i^3$ is a polynomial of n with degree 4. Namely:

$$\sum_{i=1}^{n} i^3 = a_4 n^4 + a_3 n^3 + a_2 n^2 + a_1 n^1 + a_0$$

for some constants a_4, a_3, a_2, a_1, a_0. Plug-in $n = 0, 1, 2, 3, 4$ into above equation, we have:

$n =$	LHS	$=$	RHS				
$n = 0$	0	$=$					a_0
$n = 1$	1	$=$	a_4	$+a_3$	$+a_2$	$+a_1$	$+a_0$
$n = 2$	9	$=$	$16a_4$	$+8a_3$	$+4a_2$	$+2a_1$	$+a_0$
$n = 3$	36	$=$	$81a_4$	$+27a_3$	$+9a_2$	$+3a_1$	$+a_0$
$n = 4$	100	$=$	$256a_4$	$+64a_3$	$+16a_2$	$+4a_1$	$+a_0$

Solving these equations, we have: $a_4 = 1/4$, $a_3 = 2/4$, $a_2 = 1/4$, $a_1 = 0$ and $a_0 = 0$. Thus:

$$\sum_{i=1}^{n} i^3 = \frac{n^4 + 2n^3 + n^2}{4} = \frac{n^2(n+1)^2}{4}$$

14. (0 points). Page 176, Prob 2 (b), (c), (d), (e), (f).

sol:

(b) countably infinite. $f(x) = -2x + 1$ is a bijection from Z^+ to the given set.

(c) finite.

(d) uncountable.

(e) countably infinite.

$$f(x) = \begin{cases} (2, \frac{x}{2}) & x \text{ is even,} \\ (3, \frac{x+1}{2}) & x \text{ is odd.} \end{cases}$$

(f) countably infinite.

$$f(x) = \begin{cases} 10 \cdot \frac{x}{2} & x \text{ is even,} \\ -10 \cdot \frac{x+1}{2} & x \text{ is odd.} \end{cases}$$

15. (0 points). Page 176, Prob 10.

sol:

(a) Let $A = [0, 1)$ and $B = (0, 1)$. Then $A - B = \{0\}$.

(b) Let $A = \{1, 2, \cdots, \} \cup \{x \in \mathbf{R} | x < 1\}$ and $B = \{2, 4, 6, 8, \cdots, \} \cup \{y \in \mathbf{R} | y < 1\}$. Then $A - B = \{1, 3, 5, 7, \cdots, \}$.

(c) Let $A = \{x \in \mathbf{R} | x > 1\}$ and $B = \{y \in \mathbf{R} | y > 1\}$. Then $A - B = \{x \in \mathbf{R} | 1 < x \leq 2\}$.

16. (0 points). Page 176, Prob 11.

sol:

(a) Let $A = [-1, 0]$ and $B = [0, 1]$. Then both A and B are uncountable. But $A \cap B = \{0\}$ is countable (it contains only 1 element).

(b) Let $A = (0, 1) \cup Z^+$ and $B = (3, 4) \cup Z^+$. Then both A and B are uncountable. But $A \cap B = Z^+$ is countably infinite.

(c) Let $A = [-2, 1]$ and $B = [0, 2]$. Then both A and B are uncountable. $A \cap B = [0, 1]$ is also uncountable.