# CSE462/562: Database Systems (Spring 24)

## Lecture 5: Tree Index

## 2/26/2024

University at Buffalo
Department of Computer Science and Engineering
School of Engineering and Applied Sciences

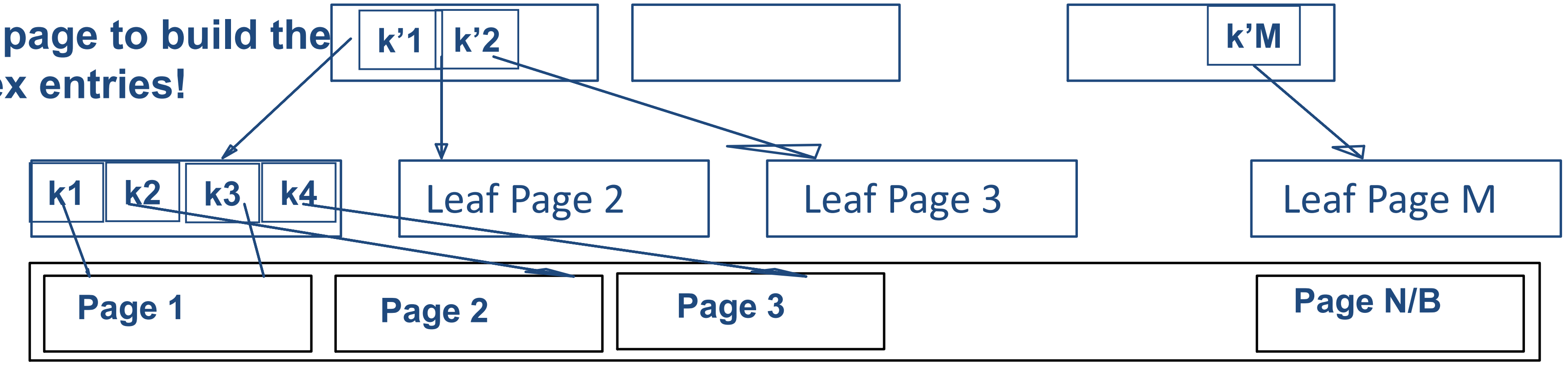
# Range Searches

- Find all the students admitted in or after 2020?
  - If data is in sorted file, we can do binary search to find the first; and then scan to find others. $O\left(\log_2 \frac{N}{B_h}\right) + scan\ cost$ -- $N$: number of records; $B_h$: number of records per heap page
  - Cost of binary search can be quite high. Hard to maintain.
- Simple idea: create an index file
  - binary search on the (smaller) index file
  - But the index file could still be quite large
    - Solution: build a new level of indirections

student

| sid | name | major | adm_year |
|---|---|---|---|
| 100 | Alice | CS | 2021 |
| 101 | Bob | CE | 2020 |
| 102 | Charlie | CS | 2021 |
| 103 | David | CS | 2020 |

**Internal pages: Take the smallest search key value from each leaf page to build the index entries!**




**Leaf Pages with Data Entries:**
**1) One data entry per record!**
**2) Sort data entries**

**Data File With Data Pages**

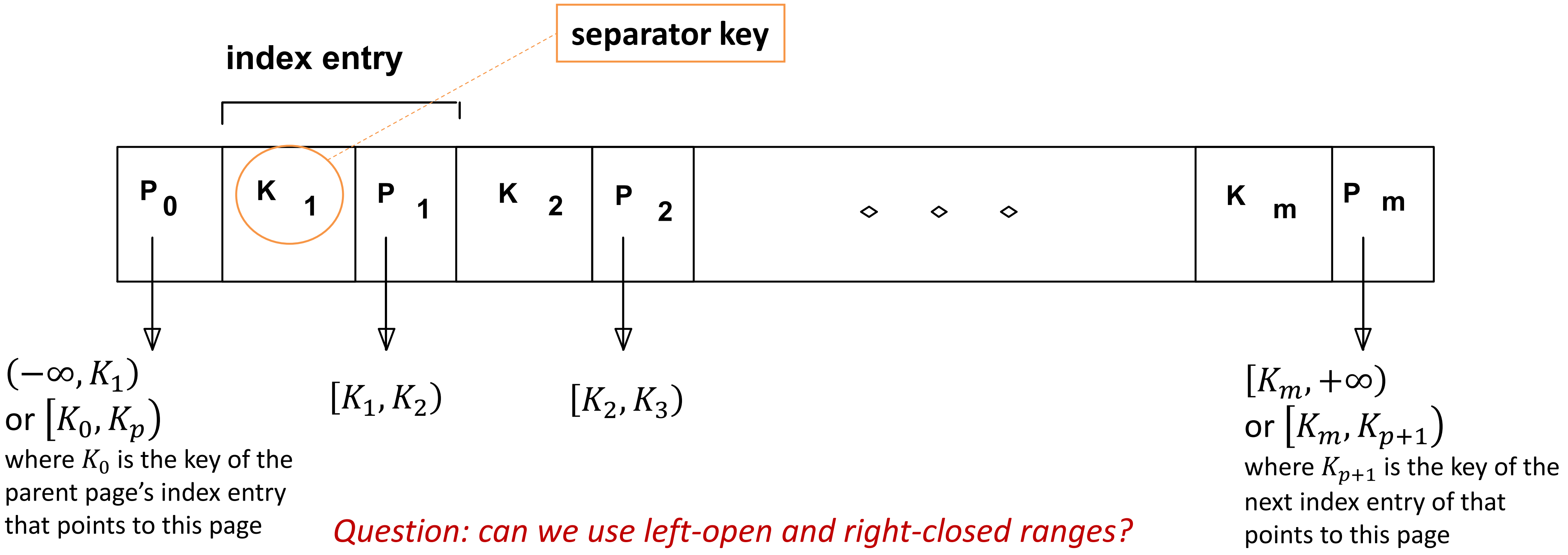
# Tree-based Indexes

- *Recall: 3 alternatives for data entries* k*:
  - Data record with key value **k**
  - <**k**, rid of data record with search key value **k**>
  - <**k**, list of rids of data records with search key **k**>
- Choice is orthogonal to the *indexing technique* used to locate data entries k*.
- Tree-structured indexing techniques support both *range searches* and *equality searches*.
- *ISAM*: static structure; *B+ tree*: dynamic, adjusts gracefully under inserts and deletes.

# Index Entries

An index entry has the following format: (search key value, page id). The following shows an index page with m index entries (pay attention to the special "left-most pointer")

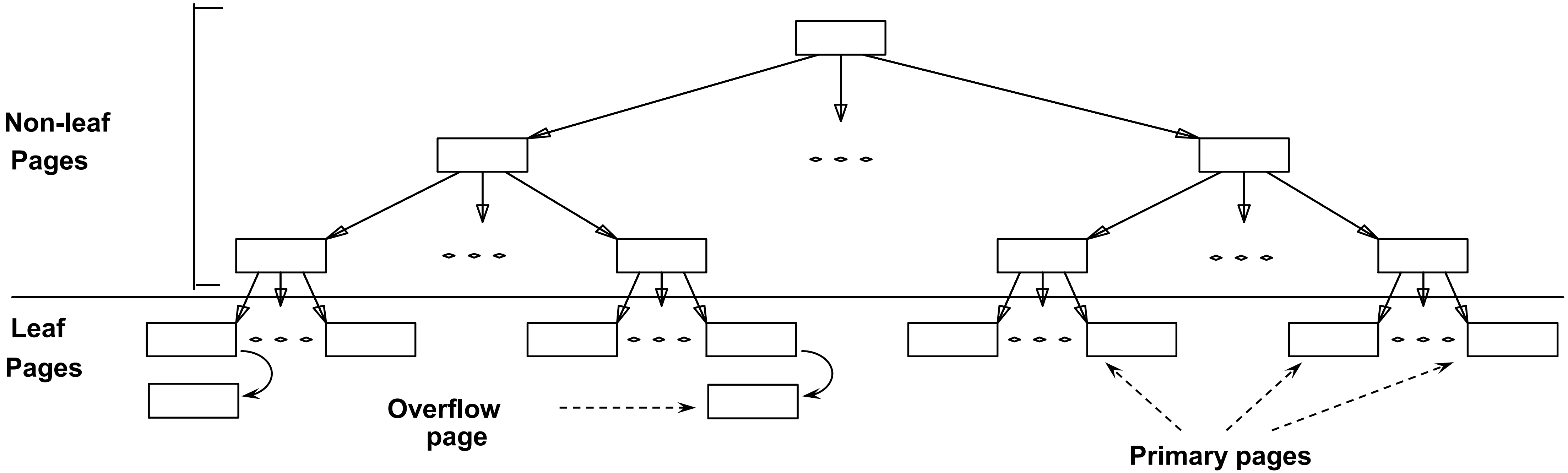
Note: entry 0 does not have a key; the range is implicitly defined by left child and K1




*Question: can we use left-open and right-closed ranges?*

# ISAM

- *Static* structure built based on the content of a heap file.
- Supports insert/delete/search.
  - Overflow pages for excessive insertions




*Leaf pages contain data entries.*

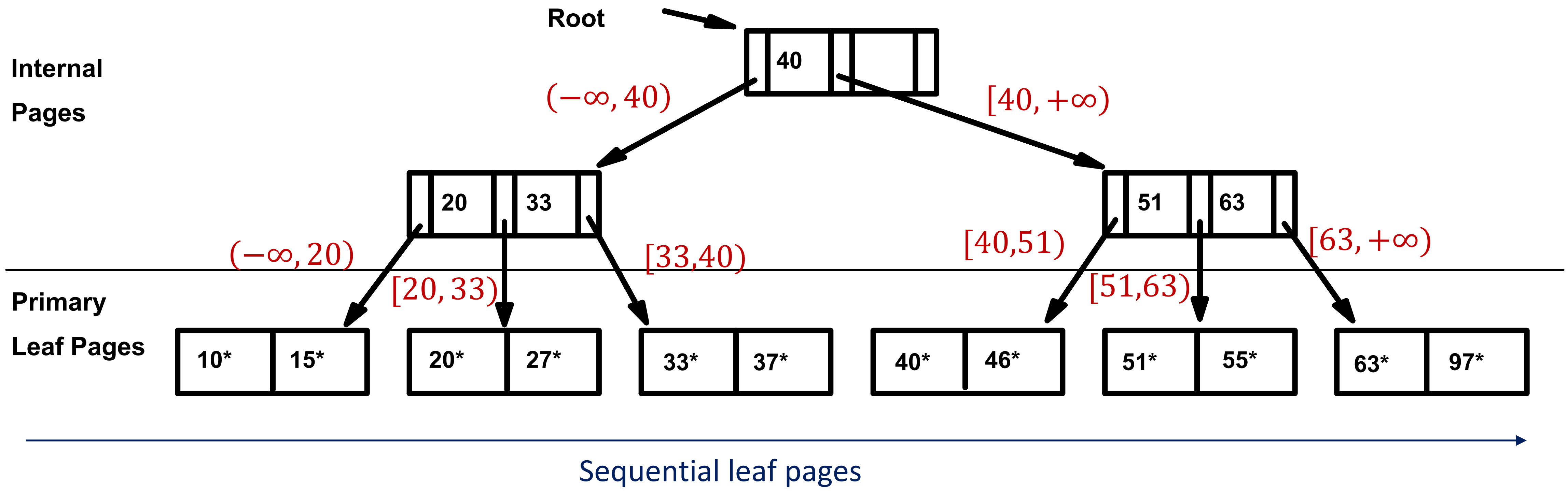
# ISAM Details

- *File creation*: With data pages in a heap file loaded.
  Leaf (data) pages allocated sequentially, and data entries sorted by search key;
  Then index pages allocated.
  Then space for overflow pages.
  - *Index entries*: <search key value, page id>; they `direct' search for *data entries*, which are in leaf pages.
- <u>*Search*</u>: Start at root; use key comparisons to go to leaf.
  I/O cost: $O\left(\log_F \frac{N}{B_0}\right)$
  F = fan-out, i.e., # entries per index page, N = # data entries, $B_0$ = # data entries / leaf page
- <u>*Insert*</u>: Find leaf where data entry belongs, put it there.
  (Could be on an overflow page).
- <u>*Delete*</u>: Find and remove from leaf; if empty overflow page, de-allocate.

- **Static tree structure**: *inserts/deletes affect only leaf pages*.
  - Not good for files with a lot of insertions/deletions
  - Could have skews/long overflow chains
- No support for variable-length records in the original ISAM design
  - MyISAM supports variable-length records, but no transaction support, no foreign-key integrity constraint support
  - In any case, you should not use ISAM in practice. But it is a good starting point for learning tree indexes.

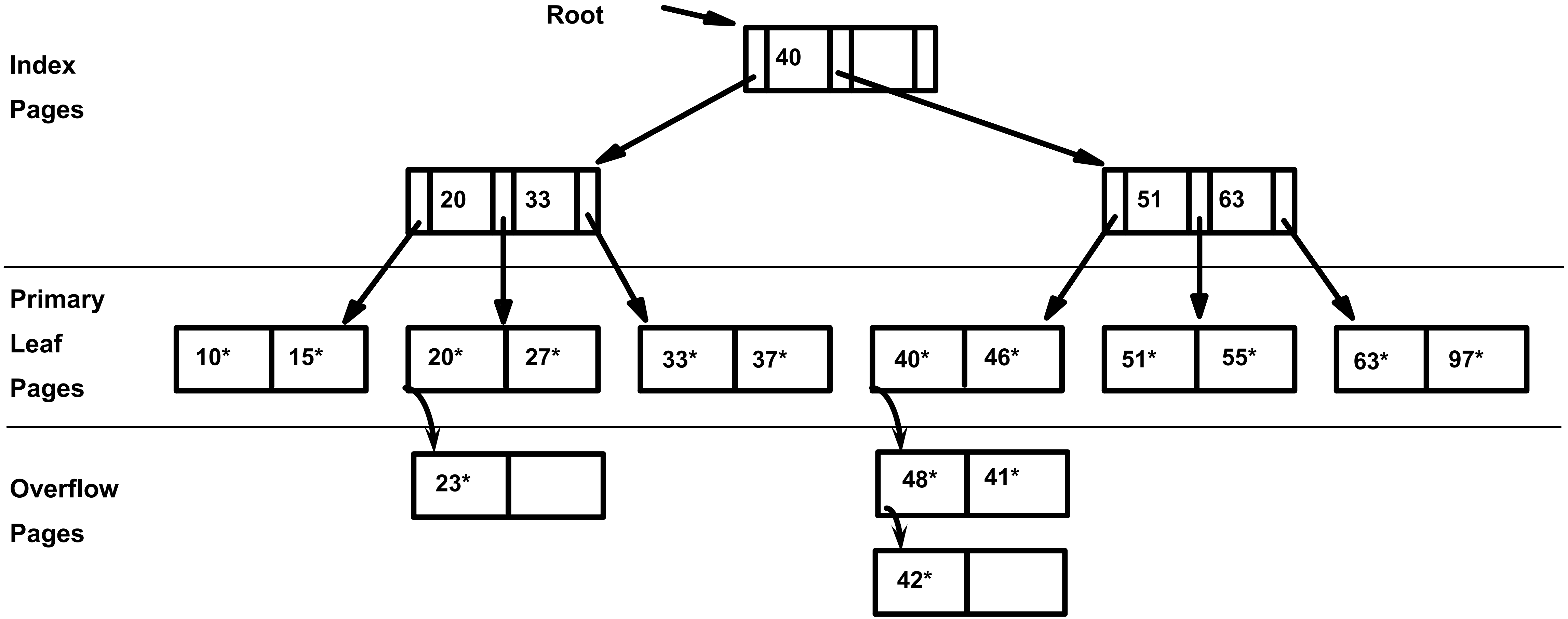
# Example ISAM

- e.g., each node can hold 2 data entries or 1 + 2 index entries
  - no need for `next-leaf-page’ pointers.  (Why?)



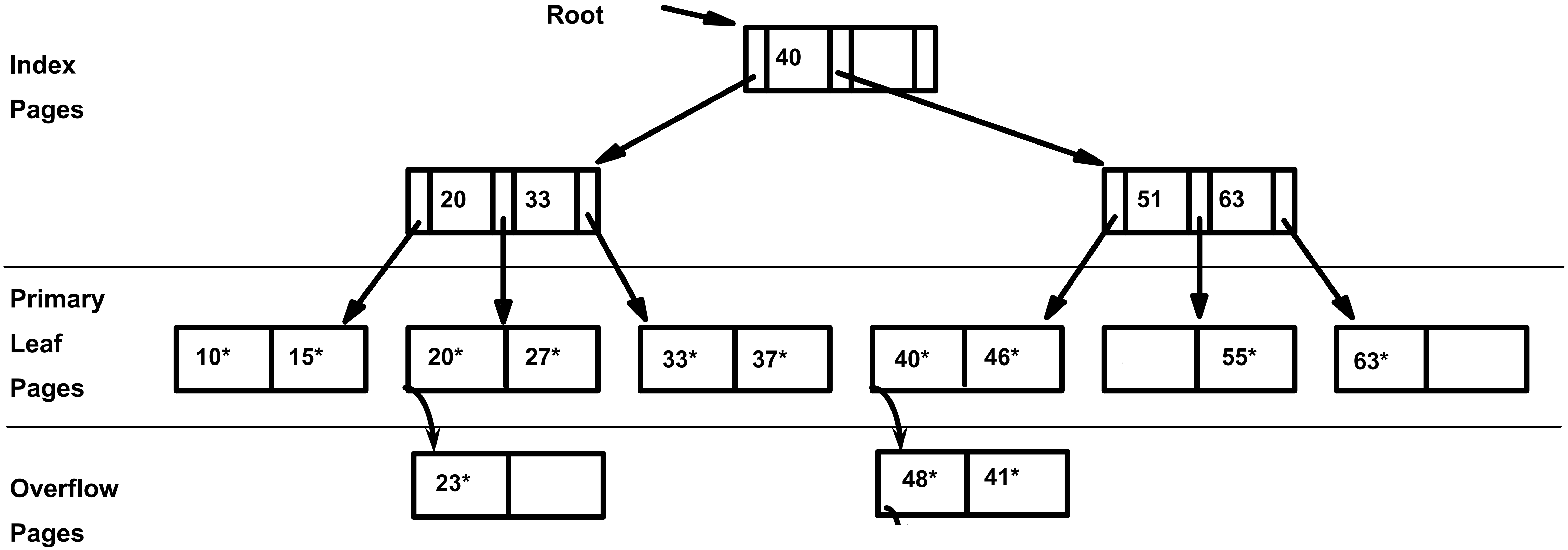
# ISAM Insertion Examples

- Inserting 23*, 48*, 41*, 42*



# ISAM Deletion Examples

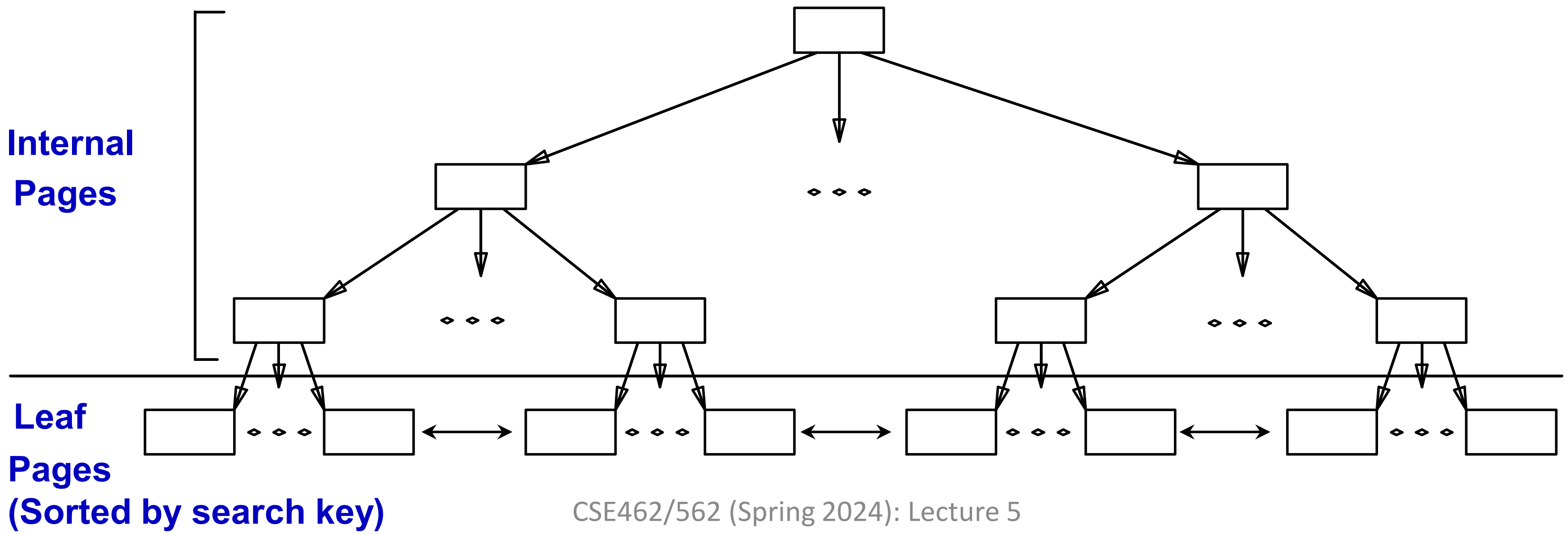
- Deleting 42*, 51*, 97*



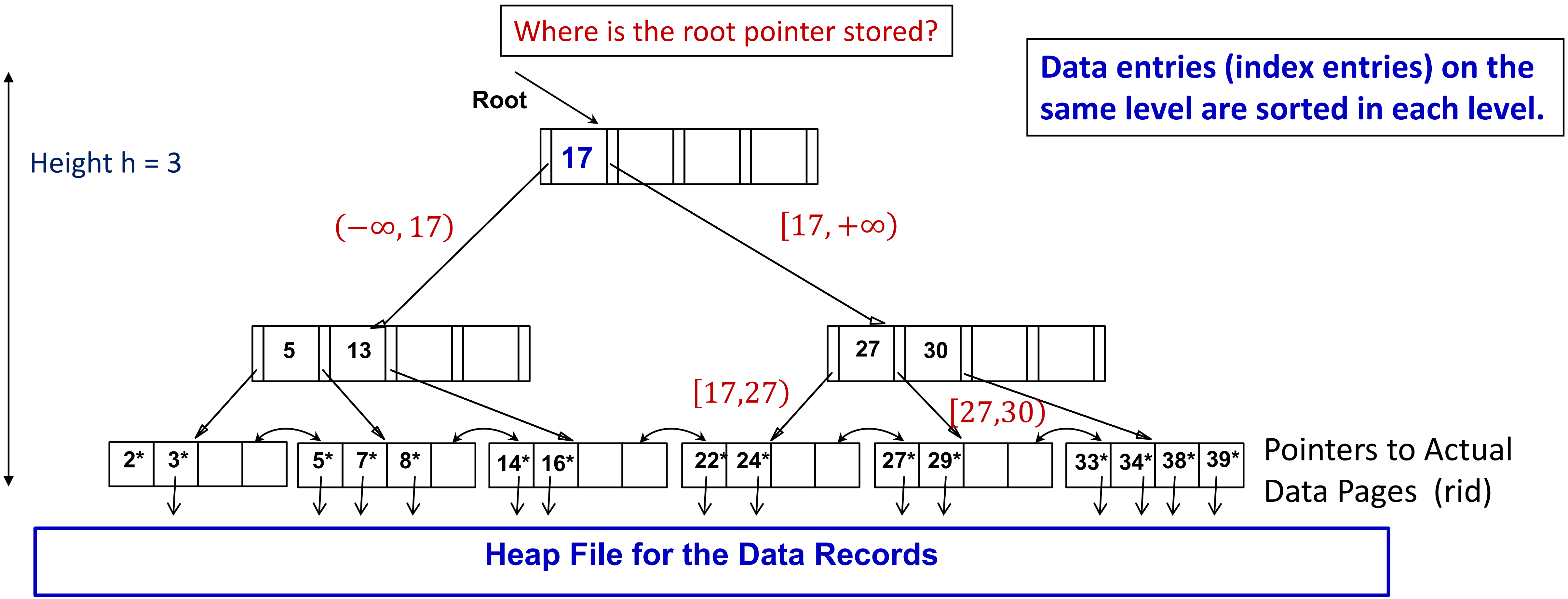

*Note that 51 appears in index levels, but 51* not in leaf!*

# B-Tree: the most widely used index

- Dynamic structure
  - Adapts to insertion/deletion
  - Data entries are stored in the leaf pages; Index entries in internal pages
  - Balanced: all paths from root to leaf page has the same length -- called tree height h
    - There's a min occupancy for each page except for root (usually 50%)
- Each node in the tree is a page in the file
  - B-Tree internal/leaf node ≡ B-Tree internal/leaf page
- Actually, it's a B+-Tree

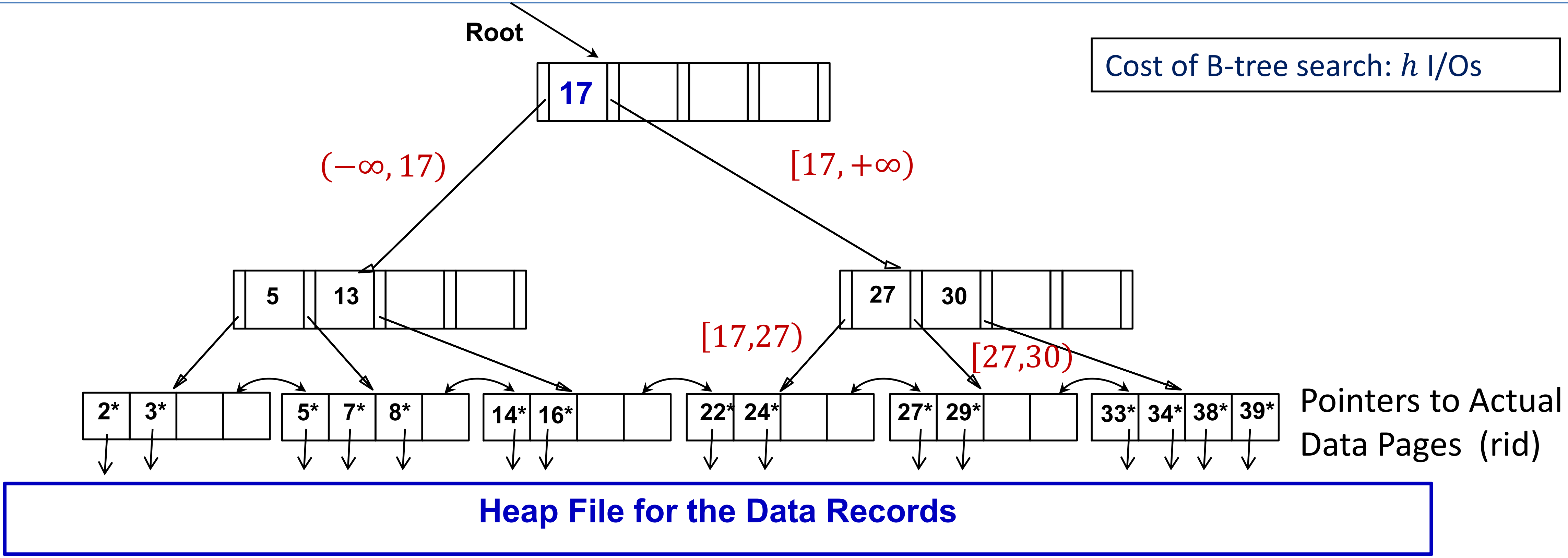


# B-Tree example




Let's assume unique and fixed-length keys for now. Leaf node capacity: $B = 4$. Fan-out $F = 5$.

# B-Tree search




Cost of B-tree search: $h$ I/Os

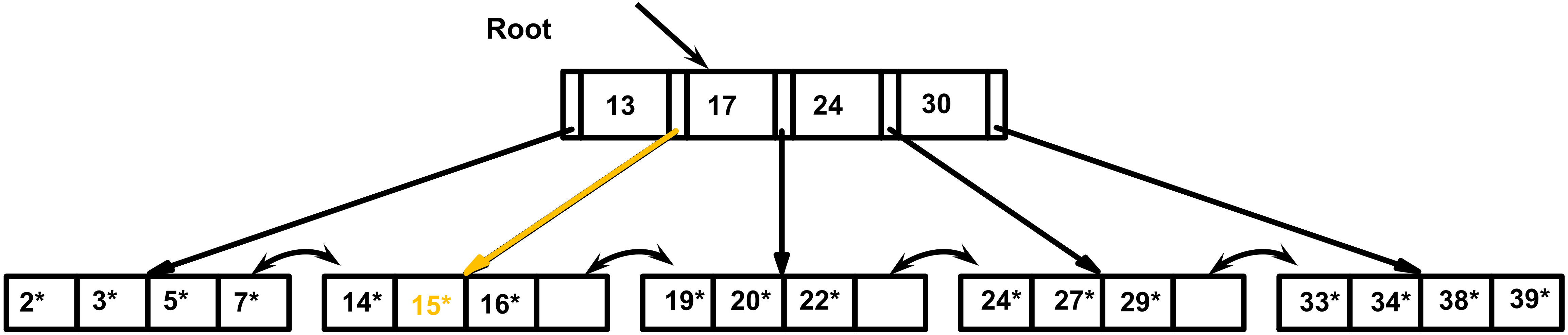
- Find 28*? 29*? All > 15* and < 30*
  - Starting from root and use key comparison to follow the correct pointers until reaching leaf.
  - To scan a range
    - Locate the lower bound of the key range
    - move right on the data entries until there're no left or you find one that's out of range
  - Can we locate the upper bound and move left instead?

# B-Tree insertion

- Find correct leaf *L.*
  - Which one? see next slide
- Put data entry onto *L*.
  - If *L* has enough space, *done*!
  - Else, must *split* *L (into L and a new node L2)*
    - Redistribute entries evenly, **copy up** middle key.
    - Insert index entry pointing to *L2* into parent of *L* with the middle key.
- This can happen recursively
  - To split index node, redistribute entries evenly, but **push up** middle key. (Contrast with leaf splits.)
- Splits "grow" tree; root split increases height.
  - Tree growth: gets *wider* or *one level taller at top.*

# B-Tree insertion example -- inserting 15*

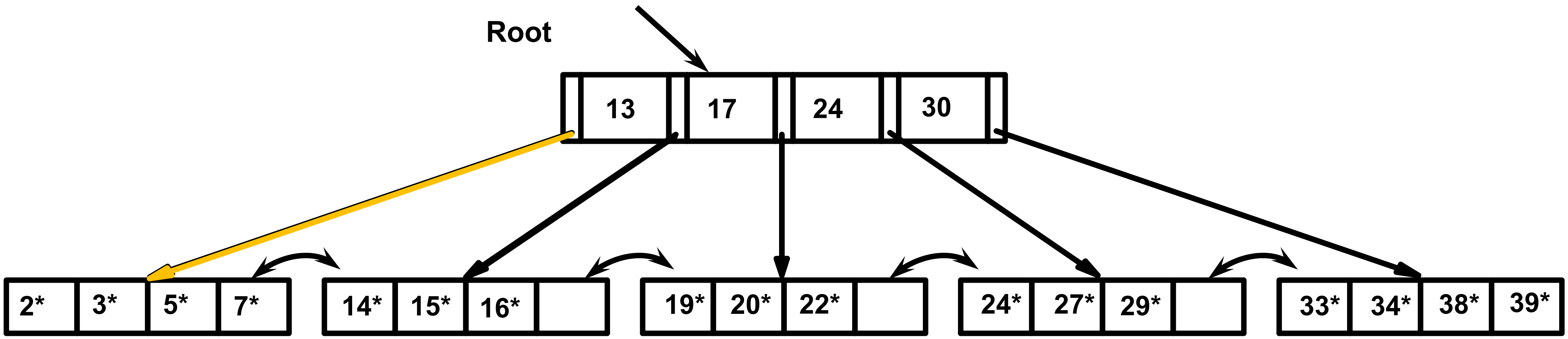
- Inserting 15*




*Find the subtree where you would do search for the insertion key.*

# B-Tree insertion example -- inserting 8*

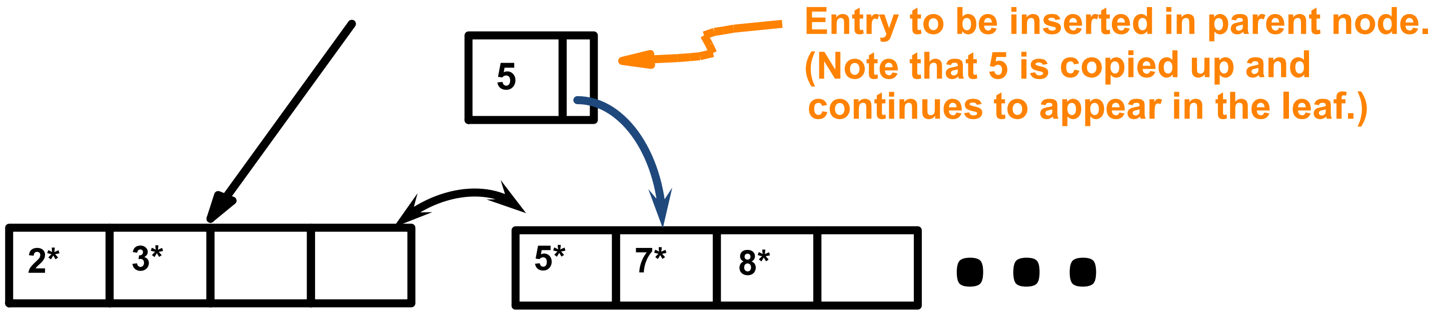
- Inserting 8*

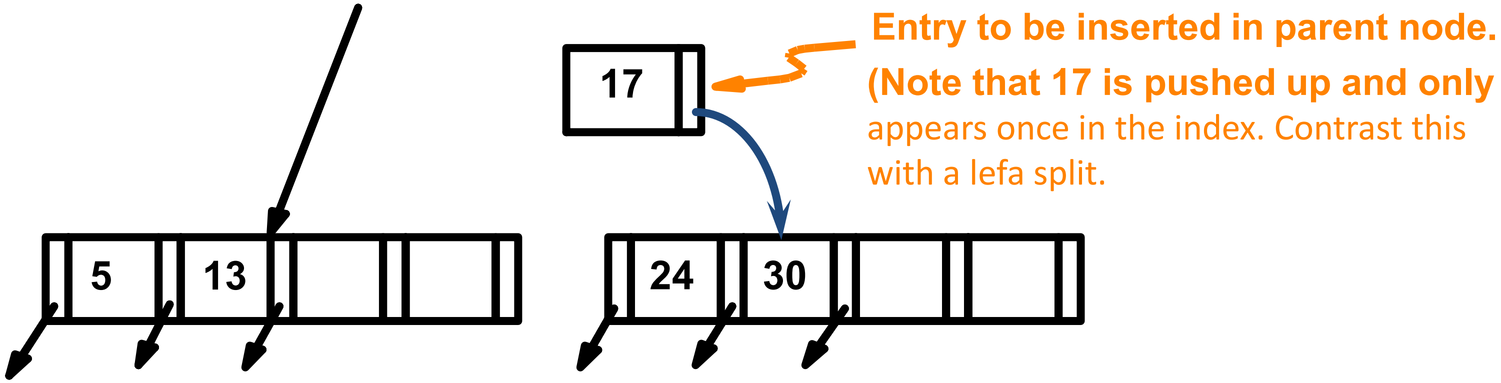



- Leaf page is full, what now? Split the page!
    - After that, the root page also needs to be split because there's no room for a new index entry

# B-Tree insertion example -- inserting 8*

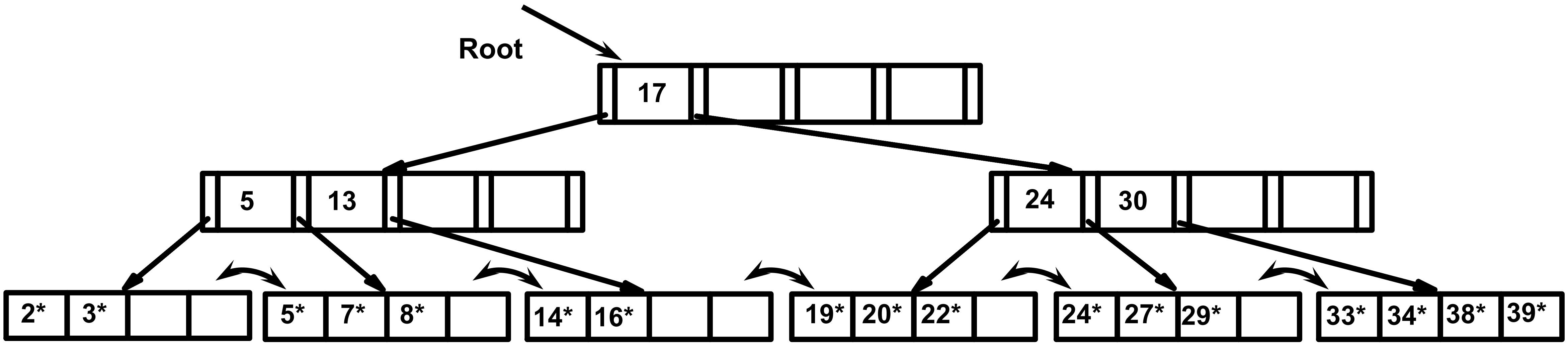
- Observe how minimum occupancy is guaranteed in both leaf and index page splits.
- Note difference between copy-up and push-up; be sure you understand the reasons for this.



# B-Tree insertion example -- Inserting 8*

Cost of B-Tree insertion: $h + 1$ to $4h + 2$ = $O(h)$ I/Os




Notice that root was split, leading to increase in height.

In this example, we can avoid split by re-distributing entries; however, this is usually not done in practice.

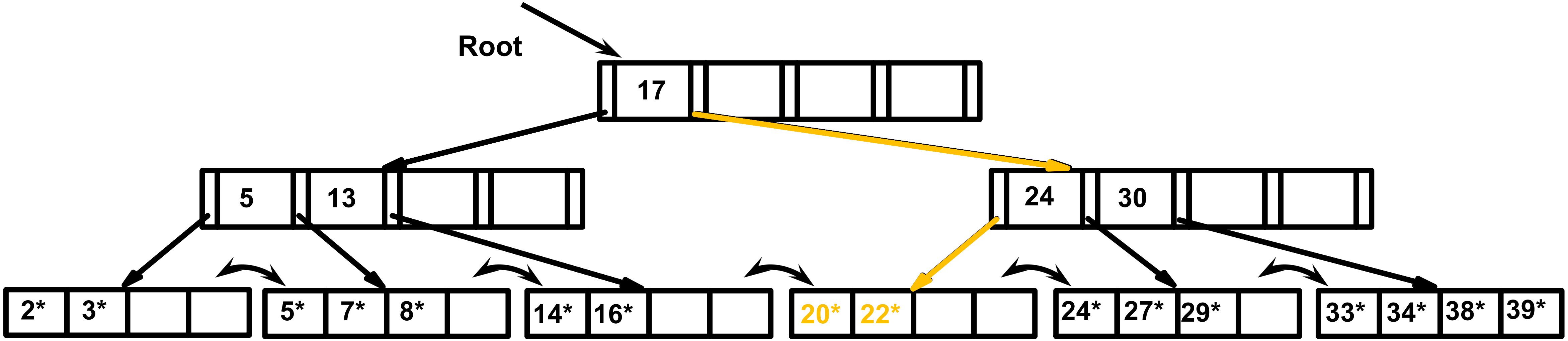
# B-Tree deletion

- Start at root, find leaf *L* where entry belongs.
- Remove the entry.
  - If L is at least half-full, *done!*
  - If L has less than half full,
    - Try to *merge* *L* and a sibling sharing a common parent.
      - Pull down the key in the parent if this is an internal page
    - Or *redistribute* keys (i.e., rebalance) between L and a sibling sharing a common parent
      - Need to update the key in the parent after rebalancing
      - Rebalancing is rarely implemented in practice, why?
- If merge occurred, must delete an index entry from parent of *L*. Which one?
  - The one on the right.
- If redistribute occurs, must update the index entry from parent of L. Which one?
  - Still the one on the right.
- Merge could propagate to root, decreasing height.

# B-Tree deletion example -- deleting 19*

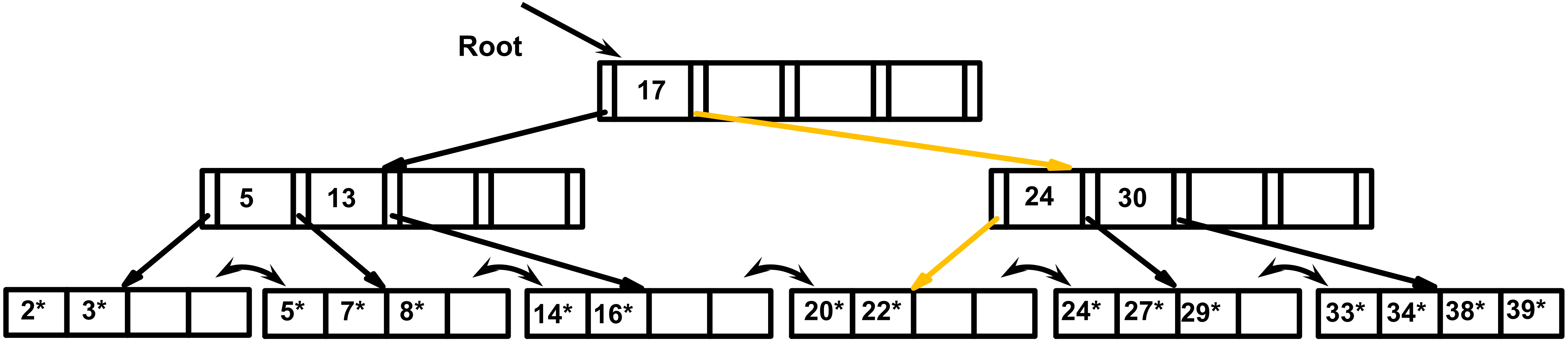
- Deleting 19* is easy.

Cost = $h + 1$ I/Os.



# B-Tree deletion example -- deleting 20* with merging

- Deleting 20* with merging. Index entry pointing the right sibling is deleted.



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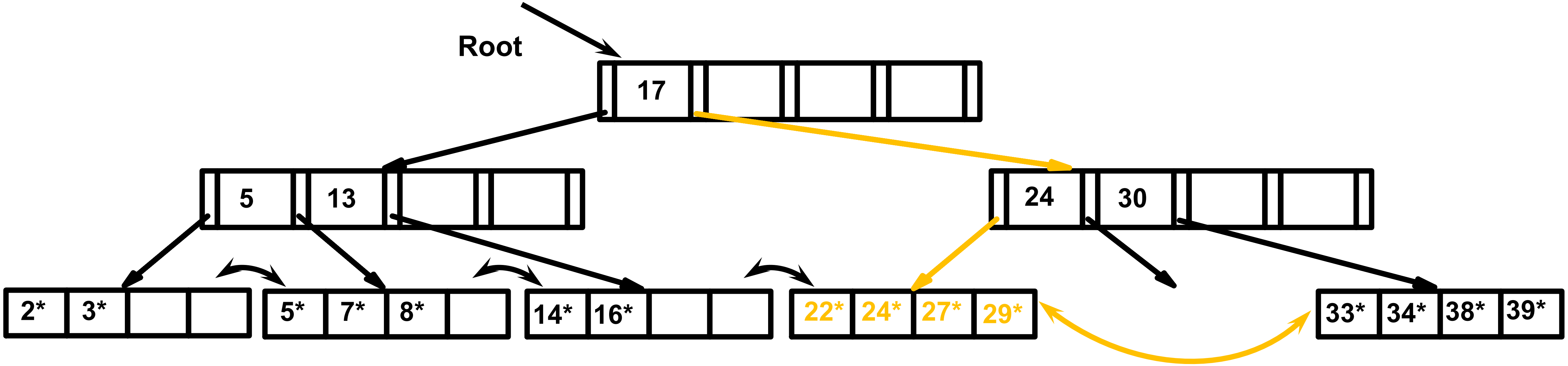
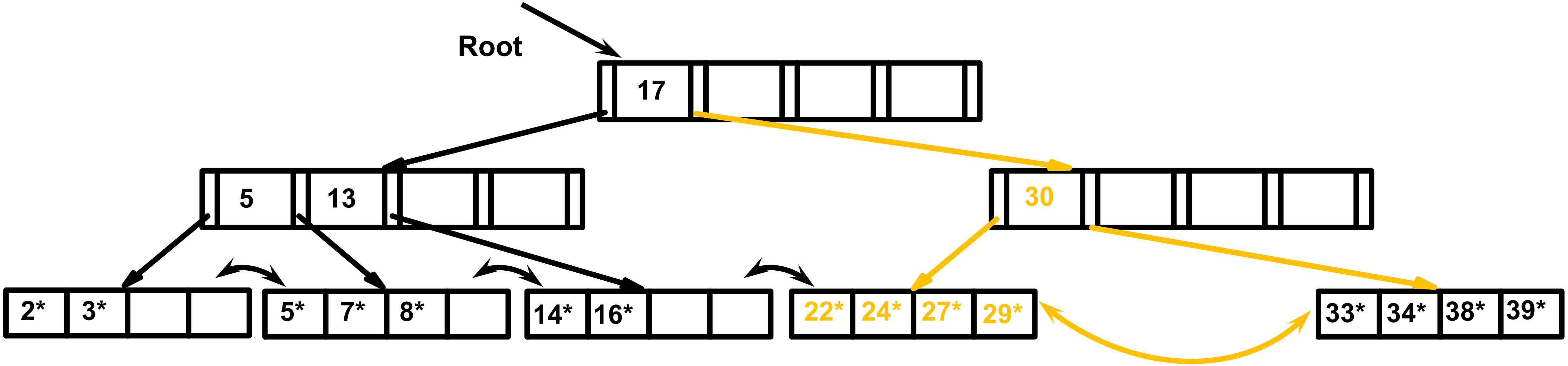


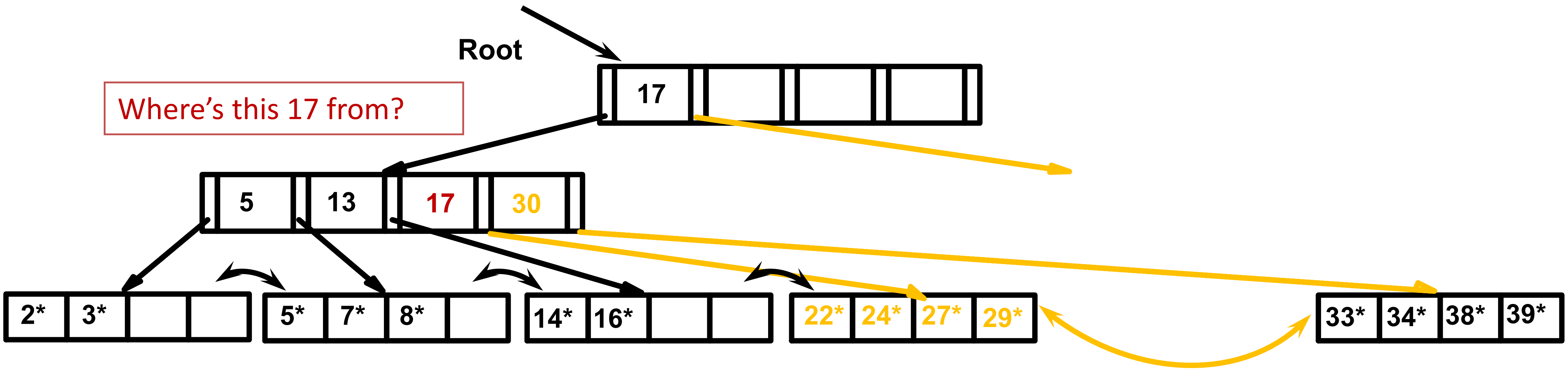
# B-Tree deletion example -- deleting 20* with merging

- Deleting 20* with merging. Index entry pointing the right sibling is deleted.
  - Internal page is also under-utilized at this point, merge it with sibling.



# B-Tree deletion example -- deleting 20* with merging

- Deleting 20* with merging. Index entry pointing the right sibling is deleted.
  - Internal page is also under-utilized at this point, merge it with sibling.
  - Root would have only one pointer at this point if we remove the index entry to the right sibling
    - need to remove the root page at this point



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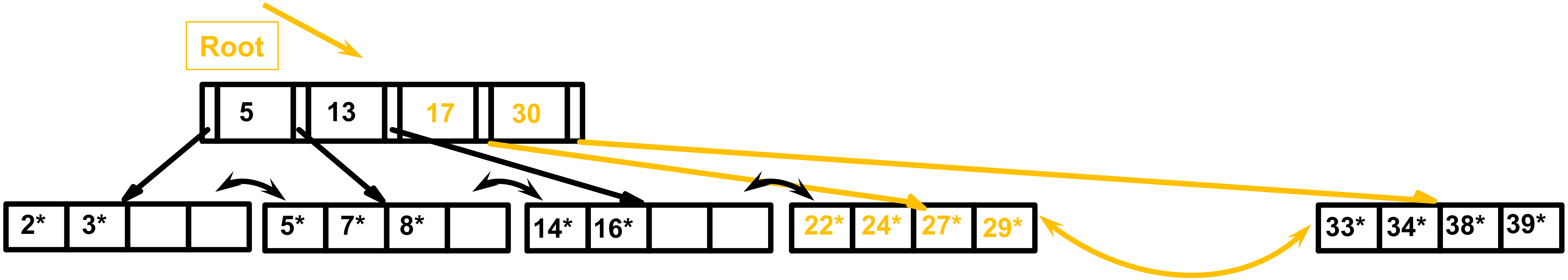
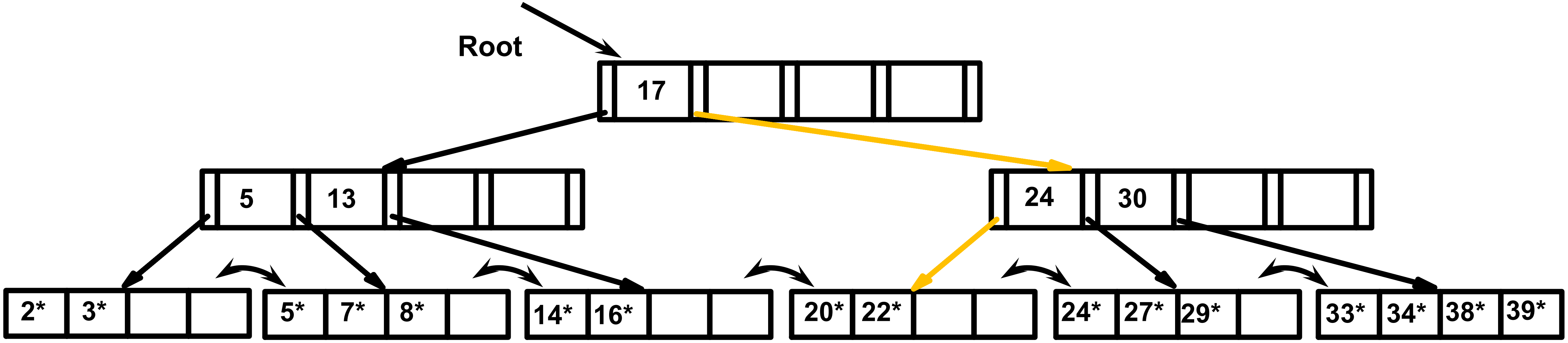
Cost = up to $4h$ I/Os.



# B-Tree deletion example -- deleting 20* with rebalancing

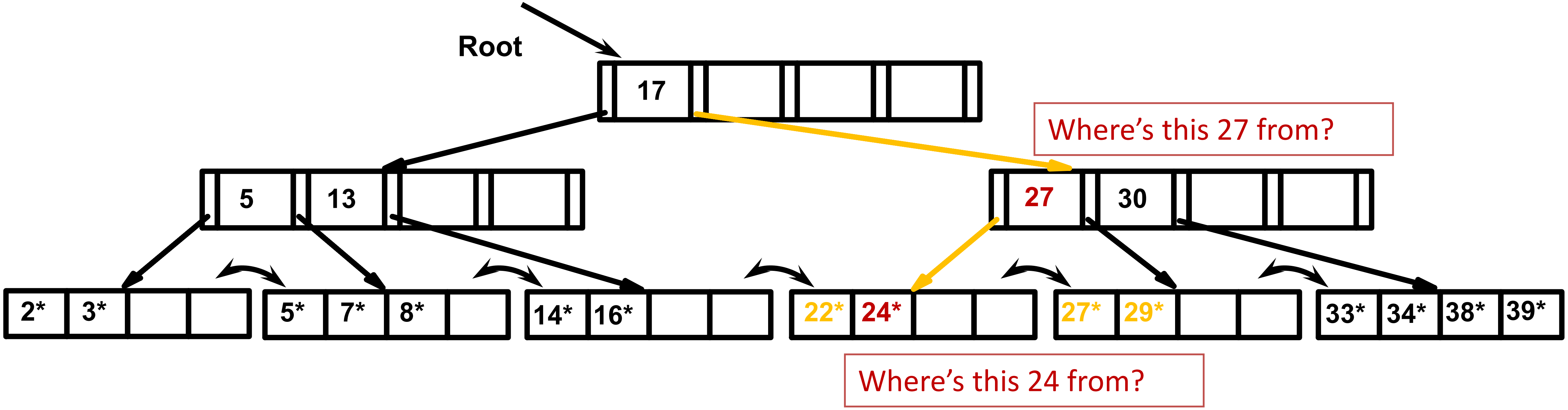
- Deleting 20* with rebalancing. Index entry pointing the right sibling is updated.
  - Copy up of the smallest key on the right page



# B-Tree deletion example -- deleting 20* with rebalancing

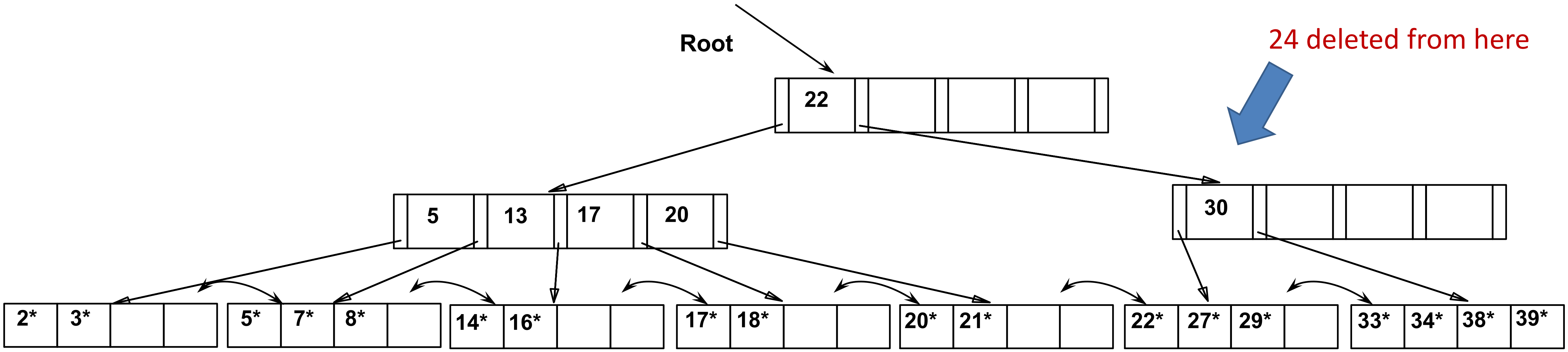
- Deleting 20* with merging. Index entry pointing the right sibling is updated.
  - Copy up of the smallest key on the right page

Cost = h + 5 I/Os.



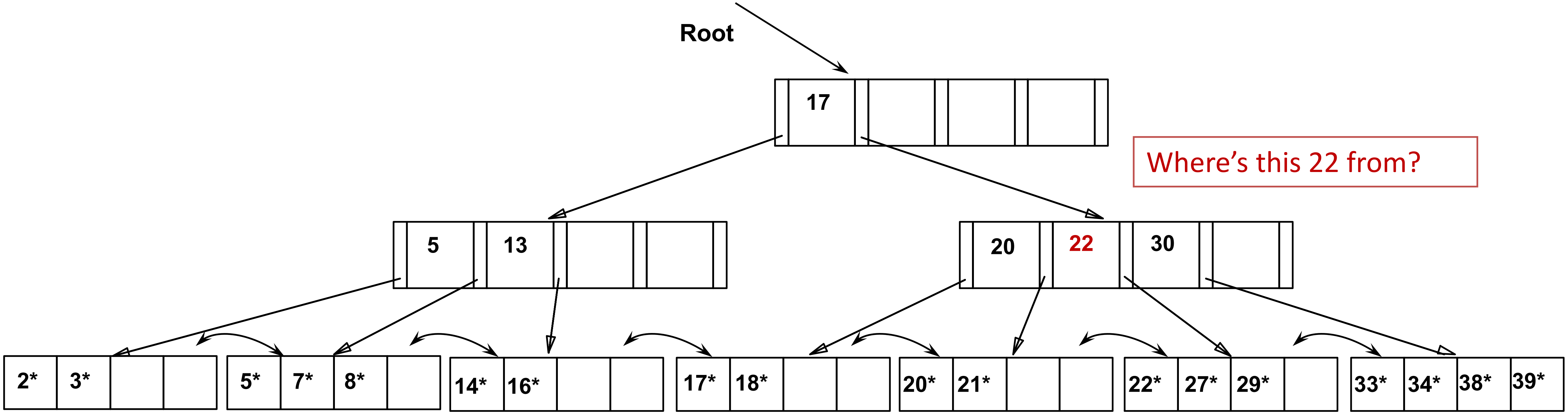
# B-Tree example of non-leaf rebalancing

- Suppose this is the tree we have and we just deleted 24* from the tree
  - which caused a deletion of an index entry on an internal page



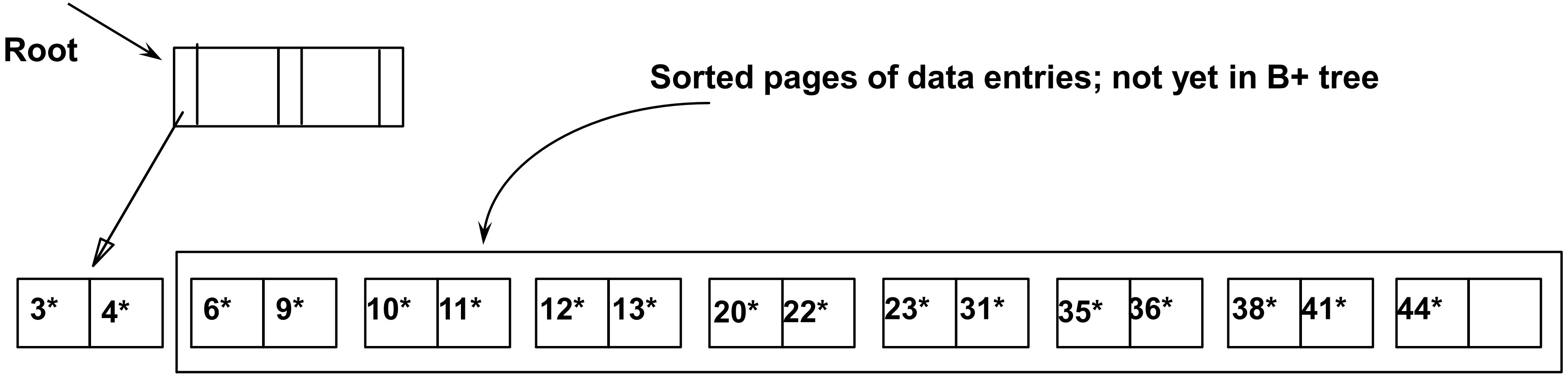
# B-Tree example of non-leaf rebalacing (cont'd)

- Intuitively, entries are re-distributed by `pushing through' the splitting entry in the parent
- Two choices: either keep 3 or 4 entries on the left page

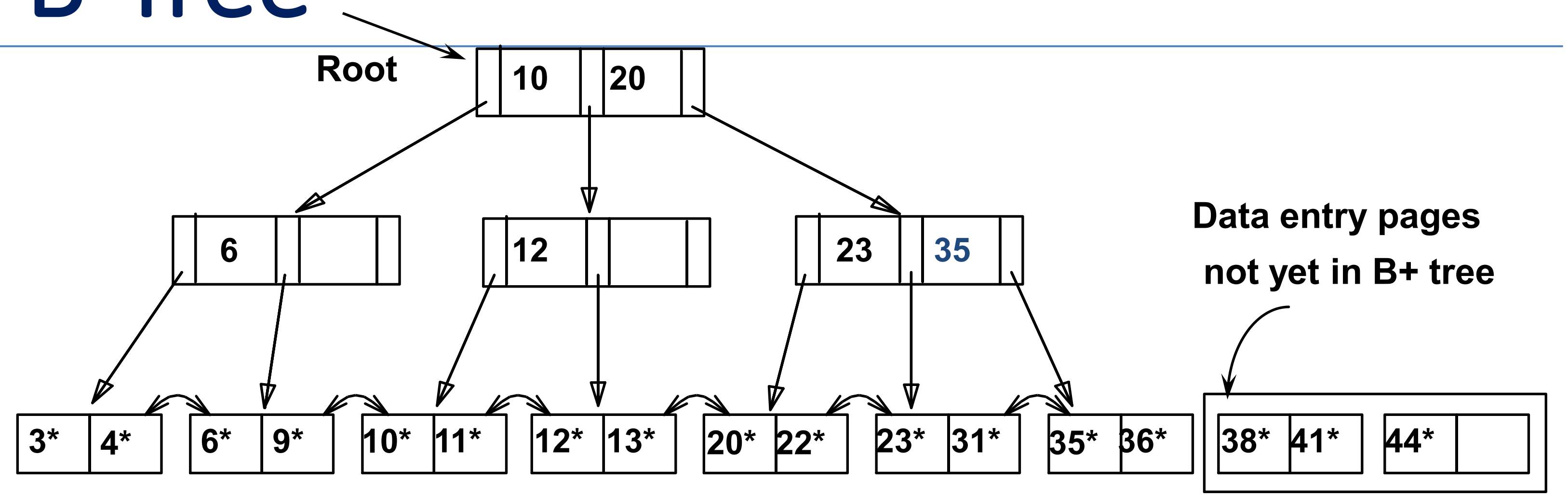


# Bulk loading of a B-Tree

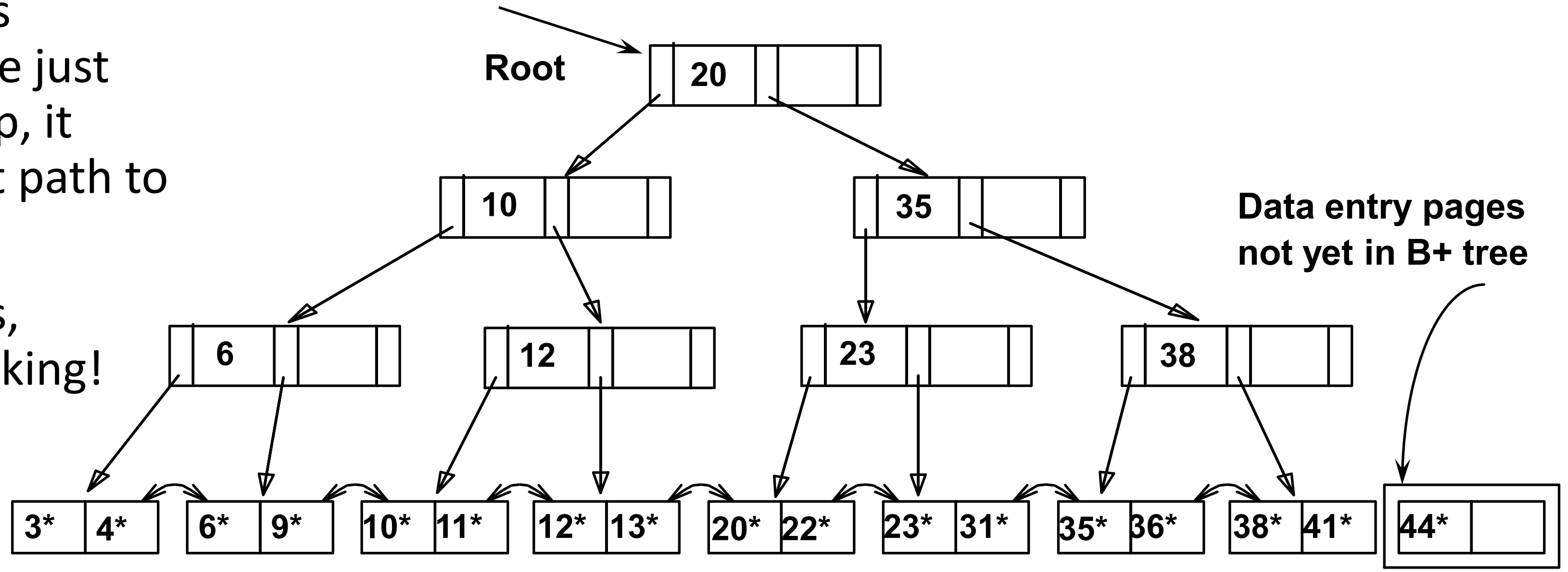
- If we have a large collection of records, and we want to create a B+ tree on some field, doing so by repeatedly inserting records is very slow.
  - Also leads to minimal leaf utilization --- why?
- *Bulk loading* can be done much more efficiently.
  - fill factor: the default utilization ratio for leaf and internal pages (may vary for leaf and internal pages) typical values: 70%/80%
- *Initialization*: Sort all data entries, insert pointer to first (leaf) page in a new (root) page.



# Bulk loading of a B-Tree




- Index entries for leaf pages always entered into right-most index page just above leaf level. When this fills up, it splits. (Split may go up right-most path to the root.)
- Much faster than repeated inserts, especially when one considers locking!



# Analysis of B-Tree storage cost

- Suppose the usable page size is $P$ (bytes), each record is $r$ (bytes), the index key is $k$ bytes, record ID or page number is $q$ bytes, and $N$ records in total in the heap file.
- Assume we use alternative 2 for the data entries.
- Bottom-up analysis:
  - Number of pages in the heap file: $M = \lceil \frac{N}{\lfloor P/r \rfloor} \rceil$.
  - Number of data entries: N (one per record)
  - Size of a data entry: $k + q$ bytes (without considering alignments)
  - Number of pages in leaf level:
    - $N' = \lceil \frac{N}{\lfloor P/(k+q) \rfloor} \rceil$
    - If the average leaf page utilization ratio is $u$:

$$N' = \lceil \frac{N}{\lfloor P * u/(k + q) \rfloor} \rceil$$

    - Let $B$ be the number of data entries per leaf page
      - $B = \lfloor P * u/(k + q) \rfloor$

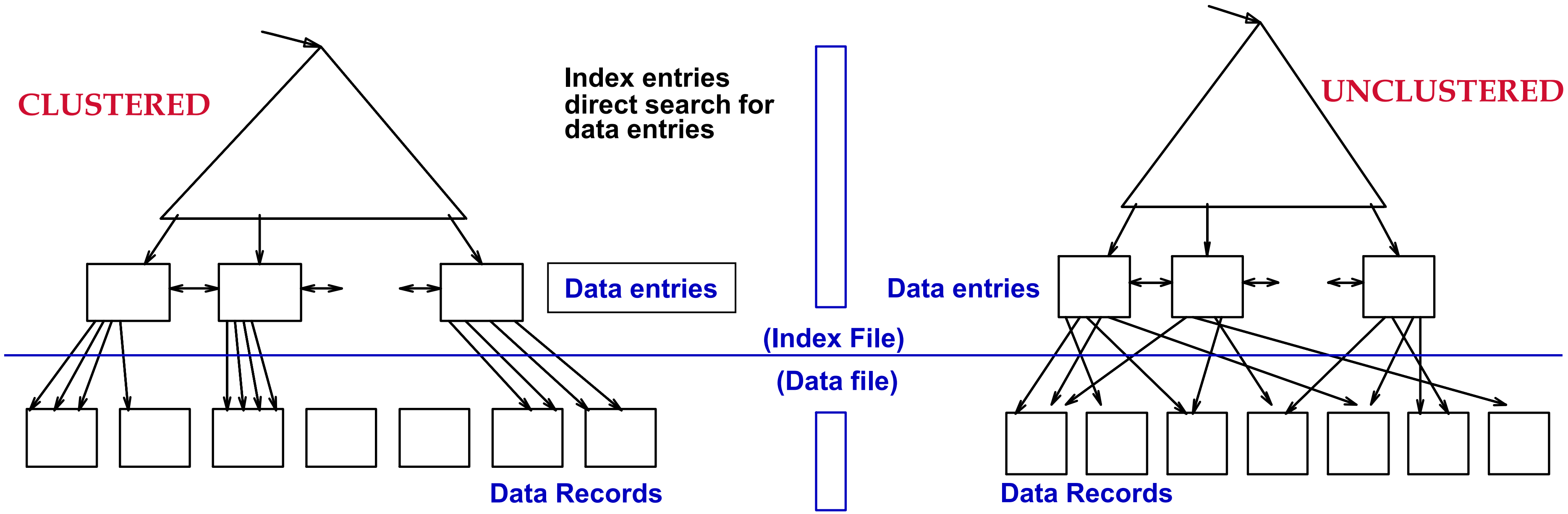
# Analysis of B-Tree storage cost

fill factor: the default utilization ratio when bulk loading the tree

- Internal levels:
  - Fan-out/number of index entries per page
    $f = \left\lfloor \frac{P \times u - q}{k+q} \right\rfloor + 1$ (u is the average utilization ratio: [0.5, 1))
  - Number of entries in the index level right above the leaf level: N' (one entry per leaf-level page)
  - Number of pages required in this level: $N'/f$
  - Number of entries in the level above: $N'/f$
  - Number of pages in the level above: $N'/f^2$
  - Recursively pages in each level:
    - N', N'/f, $N'/f^2$ , $N'/f^3$ …. $1=N'/f^{h-1}$
    - So $h = \lceil \log_f N' \rceil + 1 = \left\lceil \log_f \lceil \frac{N}{B} \rceil \right\rceil + 1$
    - total number of internal pages $1 + f + \ldots + f^{h-1} = \frac{f^h - 1}{f - 1} = O(N') = O(N/B)$
- Total number of pages in a B-Tree: $O(N') = O(\frac{N}{B})$

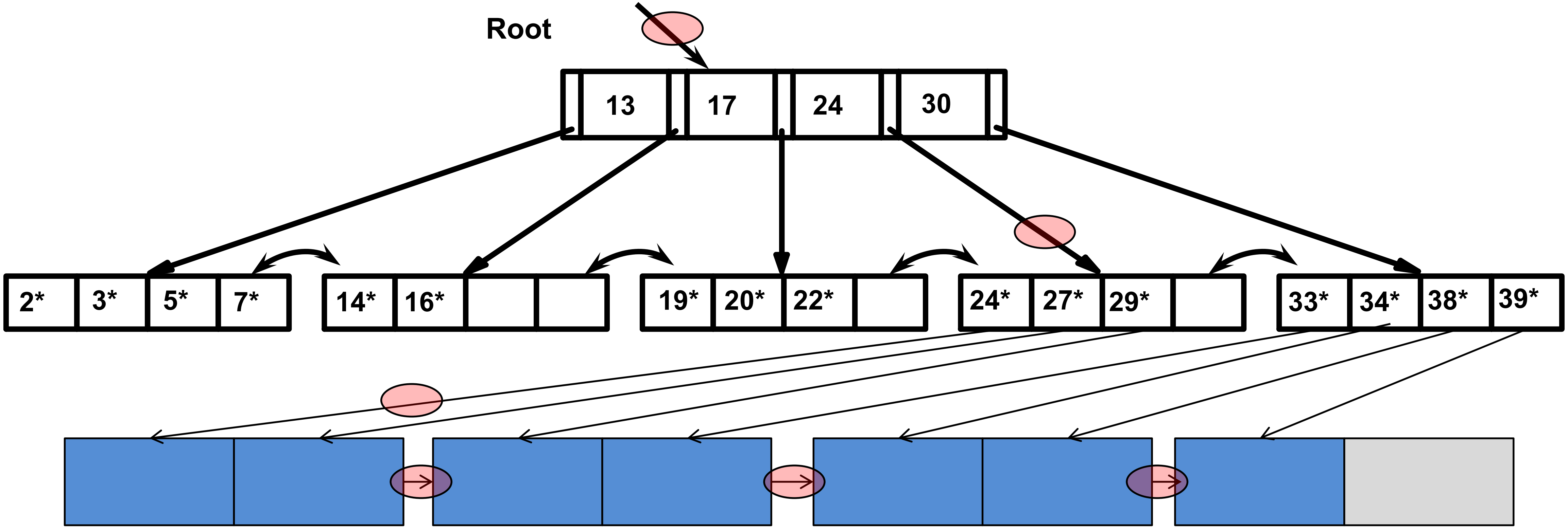
# Data access cost using B-Tree

- Recall clustered vs. unclustered: if order of data records is the same as, or `close to', order of index data entries, then called clustered index.
  - ***Cost of using B-Tree to access records varies a lot depending on whether it is clustered or not***



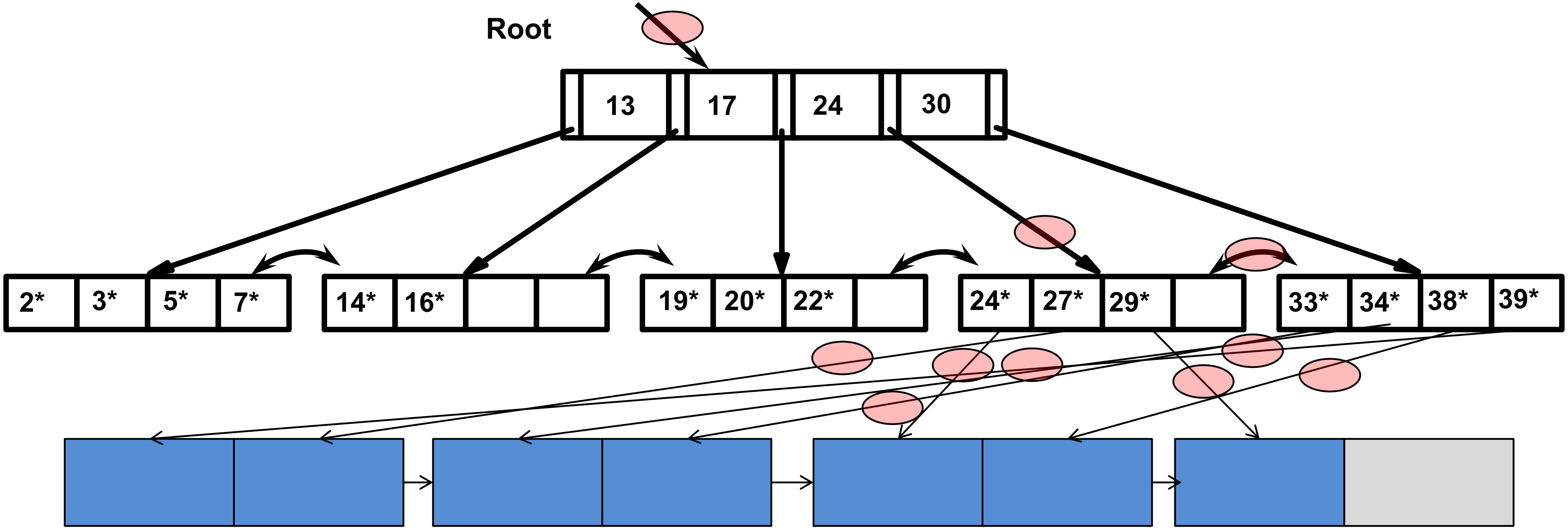
# Cost of range scan with clustered B-Tree index

- All records with key >= 24. Clustered index with alternative 2.
  - 6 I/Os
    - 2 random I/O
    - 4 sequential I/O if heap file is laid out sequentially



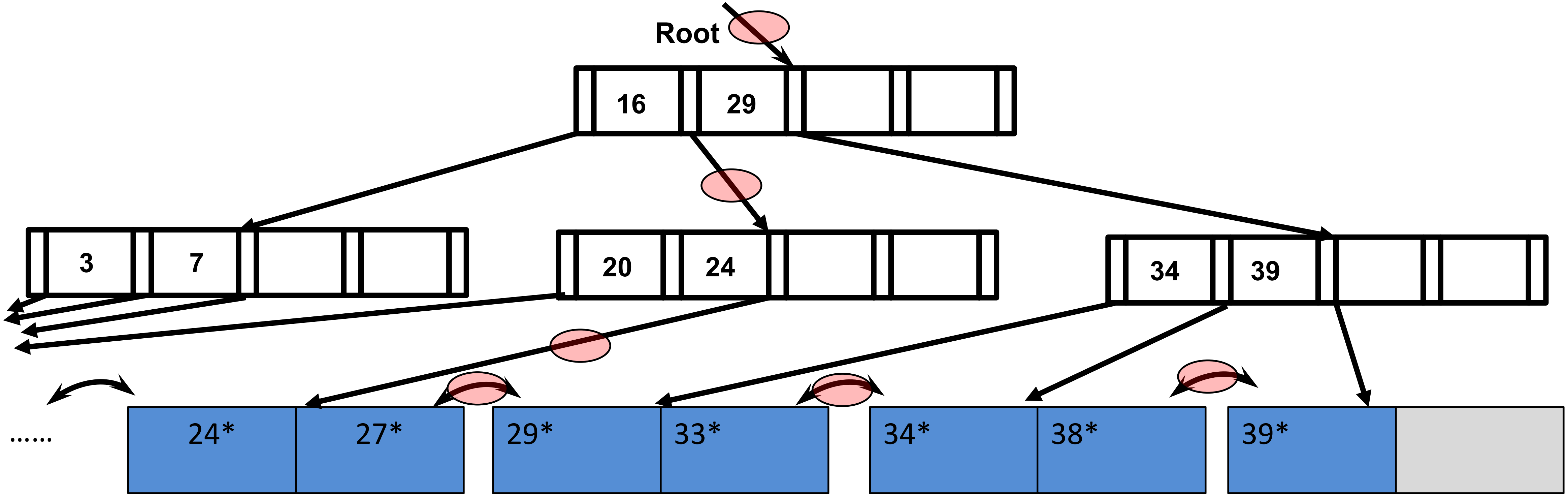
# Cost of range scan with unclustered B-Tree index

- All records with key >= 24. Unclustered index with alternative 2.
  - 10 I/Os
    - All random I/Os



# Cost of range scan with clustered B-Tree file

- All records with key >= 24. Clustered index with alternative 1.
  - 6 I/Os
    - 3 Random I/O
    - 3 Sequential I/O if the leaf level is sequential in the file



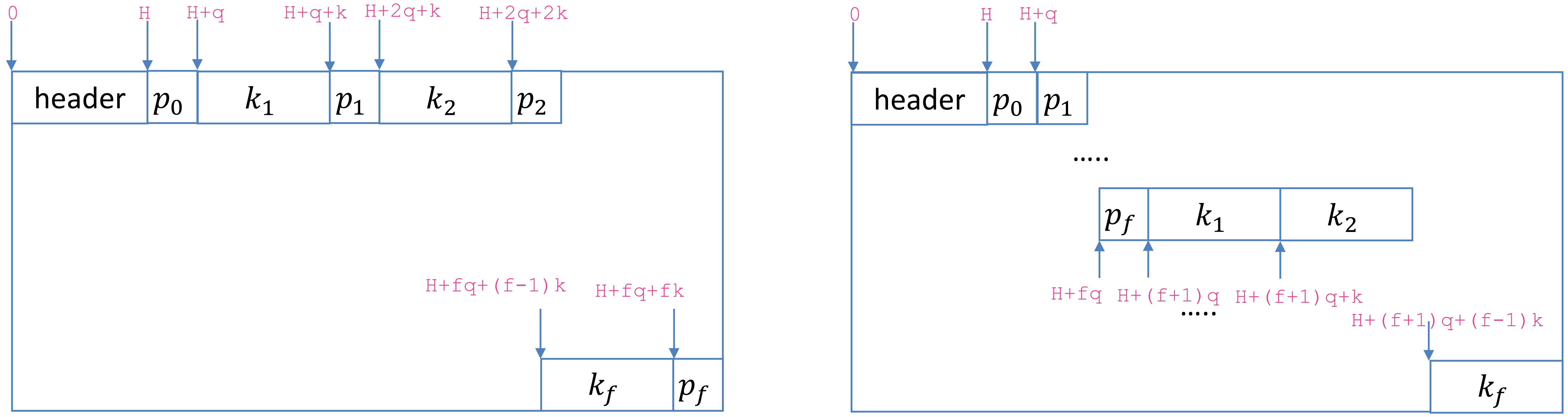
# Trade-offs with B-Tree

- Clustered B-Tree
  - One per table
  - Both are good for large range scans, small range scans and point lookups
  - Alternative 2/3 (clustered index)
    - A bit easier to maintain – can be lax on the heap record order (“close to” the data entry order)
  - Alternative 1 (clustered file)
    - Harder to maintain – strictly clustered
      - Need to reorganize the leaf level to make sure they are sequential
    - Save space on data entries (no duplication of keys)
    - Might have larger tree height

- Unclustered B-Tree
  - Usually alternative 2/3
  - Easiest to maintain
  - Not very efficient when range scan covers too many records
    - Rule of thumb: Scan no more than a tiny fraction of rows
      e.g., 0.01% on 7200 rpm HDD, 0.1% on consumer-level Nand SSD
      *(empirical value, it may vary depending on your DBMS and storage device)*

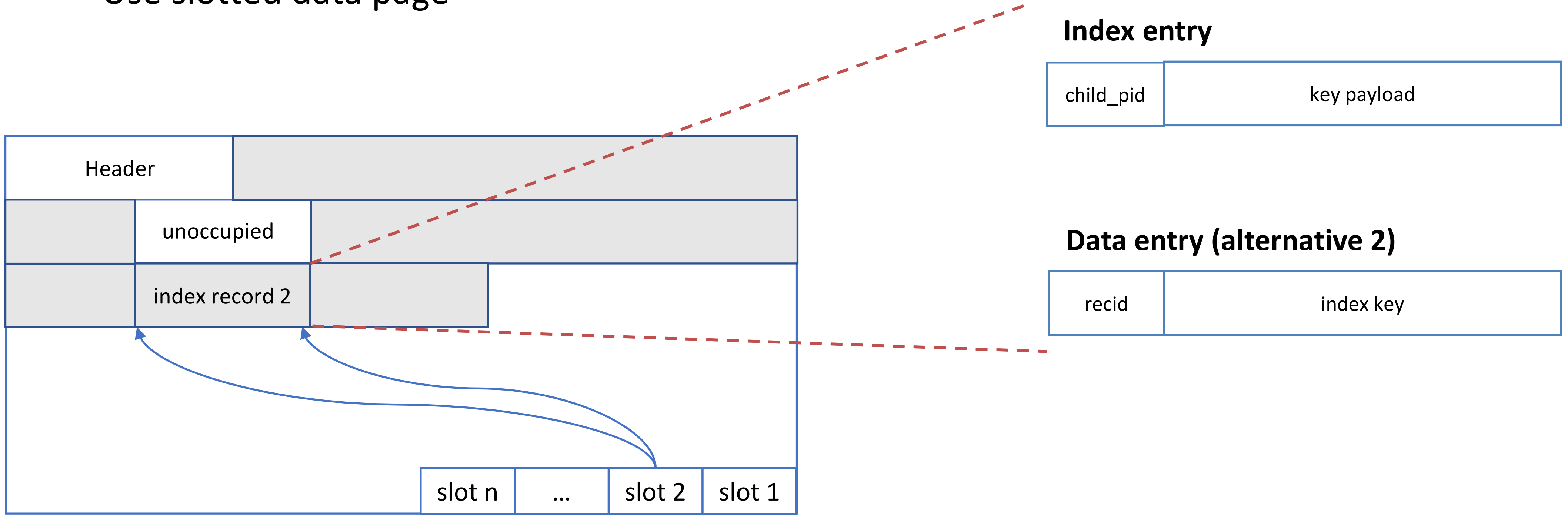
# B-Tree in practice: page and record layout

- So far, we considered fixed-length keys => fixed-fanout
  - Easy to define page occupancy in terms of number of slots
  - Easy to implement leaf and internal nodes
    - Option 1: alternating pointers and keys
    - Option 2: two arrays for pointers and keys
    - Both with fixed offsets!



# B-Tree in practice: page and record layout

- But, we could have variable-length keys
  - Nullable columns, string keys
- How do you organize the B-tree nodes?
  - Use slotted data page



# B-Tree in practice: structural modification

- How do you define page utilization?
  - How many bytes are used? How many slots there are?
    - Issues?
- Page split – that's usually ok
- Page merge
  - Leaf page merge – no problem
  - Internal page merge -- the key to pull down from the parent page may not fit!
- Page rebalance
  - Leaf or internal page rebalance
    - the key to copy/push up may not fit in the parent page!
  - Internal page rebalance:
    - the key to pull down from the parent page may not fit here!
  - Rarely implemented -- also makes concurrency control hard

# B-Tree in practice: multi-field keys

- Multi-field keys are totally ordered in the lexicographical order (aka dictionary order)
  - e.g., (a, b, c), order by a first, then b, finally c
- Multi-field keys in B-Tree is very useful
  - You can answer certain queries with predicates of a prefix of the keys
  - For instance, with a B-Tree over $(age, gpa)$, it may be used for answering the following queries:
    - $age \geq 20 \wedge age \leq 25$
    - $age = 20 \wedge gpa \geq 3.0$
    - What about $age \geq 20 \wedge gpa \geq 3.0$ ?
      - Strategy 1: using B-Tree to locate the first data entries with
        $$(age = 20 \wedge gpa \geq 3.0) \vee age > 20$$
        then scan all data entries starting from that
      - Strategy 2: for each of the distinct age >= 20, locate the first data entry with gpa >= 3.0
        then scan data entries starting from these first data entries separately
        (aka index skip scan (e.g., Oracle) /jump scan (e.g., DB2) in various systems)
        *Strategy 2 only works when there are few distinct values in the prefix column*

# B-tree in practice: NULL values

- We need to index NULL values in B-tree indexes
  - because indexed columns may have NULLs
- Caveat: SQL 3-value logic
  - *NULL < anything is unknown!*
  - B-tree requires a total order of the key
- Solution: don't use the SQL 3-value logic
  - *For instance, define NULL = NULL, NULL < any non-NULL value*
  - *Alternatively, NULL = NULL, NULL > any non-NULL value*
  - Some systems support both
  - In the course project Taco-DB, we assume NULL < any non-NULL value for indexing

# B-Tree in practice: non-unique keys

- So far, we assumed unique keys, but
  - we might create indexes over non-unique columns (e.g., name)
- B-Tree can be modified to support duplicate keys, but
  - How do you find the data entry for a specific record for update?
- What if we still want to uniquely identify keys in the tree?
  - Include record ID as the last column
    - record IDs are always unique
  - Then a search with key in B-Tree only becomes prefix search:
    - e.g., key = (age, gpa), actual key = (age, gpa, record id)
    - Query: $age = 22 \land gpa = 3.7$?
      - Locate the first data entry such that $(age = 22 \land gpa \geq 3.7) \lor age > 22$
      - Then scan the data entries until it falls out of range
  - To uniquely locate a data entry for a record: use the full search key

# B-Tree in practice: unique constraints

- B-Tree are often used for enforcing UNIQUE constraints
  - e.g., `sid SERIAL PRIMARY KEY`
  - e.g., `login VARCHAR(20) UNIQUE`
- Build unique B-tree index
  - Reject insertion of a data entry whose key already exists in another data entry in the index
    - even if the record id does not match
- However, what about NULLs?
  - Nullable unique column is allowed to contain multiple NULLs (because they are unknown values)
  - Reality: some allow and some don't
    - Some DBMS disallows inserting multiple NULLs into unique B-Tree index
      - non-conformant to SQL, but easier to implement (no special case handling)
    - Some do allow that
      - SQL-conforming, but need special handling logic for that

# B-Tree in practice: handling concurrency

- Lock-based (e.g., reader-writer lock, in DBMS jargon: latches)
  - Many issues:
    - Should lock at most c pages at a time (c usually is 1/2/3)
    - Lock coupling order (deadlock avoidance)
    - Insertion:
      - Split will cause key space shift (how does concurrent search handle this?)
      - Root split? How to install the new root with concurrent readers?
    - Deletion (harder):
      - Page merge/reducing tree height: also causes key space changes
        - Some design avoids them by deleting a page only when it's completely empty
        - Some design use mini transactions to handle SMO
    - File space management:
      - What if a page is deleted but a concurrent reader reaches the deleted page?
    - Recovery: what if crashes and we have to roll back a half completed B-tree update?
- Lock-free
  - Using CAS and additional indirection (J. Levandoski, D. Lomet, S. Sengupta. ICDE '13)
  - Other considerations?

# B-Tree in practice: key compression

- We want high fan-out → low tree height → faster query/update
  - But string keys are often quite long (tens of bytes vs 4 bytes/8 bytes)

- Prefix key compression: extract the common prefix and only store the unique suffix
  - Sorted keys tend to have a short common prefix

| | | |
|---|---|---|
| Compute | Compression | Compile |

→

| | | | |
|---|---|---|---|
| Comp | ute | ression | ile |

- Suffice truncation: store only the prefix that is enough for differentiating the subtree range
  - Works for both string/multi-field keys

| | | |
|---|---|---|
| Dannon Yogurt | David Smith | Devarakonda Murthy |

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| | | |
|---|---|---|
| Dan | Dav | Dev |

| | | |
|---|---|---|
| (1,5) | (2,3) | (2,4) |

→

| | | |
|---|---|---|
| (1, NULL) | (2,NULL) | (2,4) |

# Summary

- This lecture
  - ISAM
  - B-Tree index
    - How to search and scan/insert/delete in B-Tree
    - Analysis of B-Tree index/file
    - B-Tree in practice

- Next lecture
  - More on indexing & cost analysis

- Reminders
  - HW2 due this Sunday (3/3, 23:59 PM EST)
  - Come to office hours (or schedule ad-hoc ones) for project 2 questions
  - Project 3 will start next Monday 3/4 ---- *start very early!*